Name:

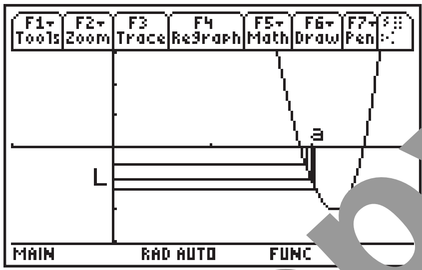
Limits: *Informal Definition of the Limit*

In this chapter, you will look at limits numerically and graphically, and learn some algebraic techniques to evaluate a limit. An informal definition of the limit is given below. You will investigate the formal definition in the last lesson of this chapter.

The **goal** is to understand the definition of the limit of a function at a finite number, *a*.

Informal Definition of the Limit, *L*, of a function, *f*(*x*), as *x* approaches a finite number, *a*.

As the value of *x* gets closer and closer to, but not equal to,
a finite number, *a*,
the values of *f*(*x*)
stay within a smaller and smaller range of the limit, *L*.



In this lesson, you will investigate the concept of a limit as *x* approaches a finite number, *a*. Note that *a* is an *x*-value and that *L* is a *y*-value.

A. Exploring Limits Numerically

You will explore the limit of a function in three ways:

- as *x* approaches a constant value, *a*, from the left,
- as *x* approaches *a* from the right, and
- as *x* approaches *a* from both directions.

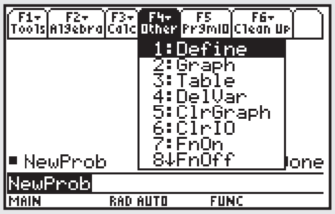
Example: Let $f(x)=\dfrac{(x-3)\cdot(x-2)}{(x-3)}$, $a=3$

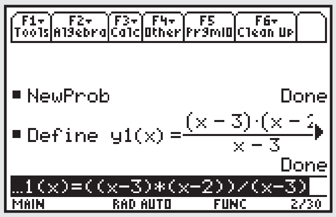
As in all investigations, begin with the **NewProb** command.

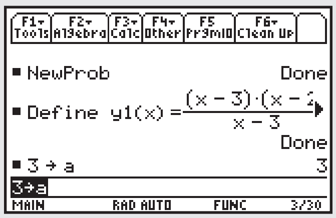
TECH-TIPS **Defining a Function, Storing a Value**

From the HOME screen:

- Press F4 1 to select Define.
- Define y1 as *f*(*x*) by entering the expression y1(x) = $\dfrac{(x-3)\cdot(x-2)}{(x-3)}$.
- Press ENTER.
- Press 3 STO▸ alpha = ENTER to store the given numerical value, 3, as *a*.







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1. Enter and evaluate the function: y1(*a*+*d*)| *d* = -0.5

Fill in the first six values in the table by editing the value of *d* as shown below. Notice the function values as *x* approaches 3 from the left.

When 3 + *d* < 3, or as *x* approaches 3 from the left, you are adding "smaller and smaller" negative values of *d*, that is $|d| \to 0^-$. This suggests that you are finding the limit of *f*(*x*) from the left.

x	$a-.5$	$a-.3$	$a-.1$	$a-.01$	$a-.001$	$a-.0001$	a	$a+.0001$	$a+.001$	$a+.01$	$a+.1$	$a+.3$	$a+.5$
y													

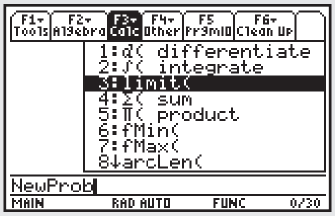
2. Make and test a conjecture: As $x \to a$ from the left, $f(x) \to$?

a. Make a conjecture of the value of the left-handed limit. $\lim_{x \to 3^-} f(x) =$ ______________________________

b. Test your conjecture by evaluating the limit on the Home Screen. Was your conjecture correct? ____________________

TECH-TIPS Evaluating the Limit

From the [HOME] screen:



- Press [F3] [3] to select limit.
- Complete the entry so that it reads limit(y1(x), x, a, -1).
- Press [ENTER].

The last parameter on the limit command is optional.

Any negative number, usually -1, indicates the limit from the left, $\lim_{x \to a^-} f(x)$.

Any positive number, usually +1, indicates the limit from the right, $\lim_{x \to a^+} f(x)$.

Excluding the last parameter indicates the limit from both directions, $\lim_{x \to a} f(x)$.

3. Enter and evaluate the function: y1(*a*+*d*)| *d* = 0.5

Fill in the last six values in the table above by editing the expression using smaller and smaller positive values of *d*. Notice the function values as *x* approaches 3 from the right.

When 3 + *d* > 3, or as *x* approaches 3 from the right, you are adding smaller and smaller positive values of *d*. This suggests that you are finding the limit of *f*(*x*) from the right.

4. Make and test a conjecture: As $x \to a$ from the right, $f(x) \to$?

a. Make a conjecture of the value of the right-handed limit. $\lim_{x \to 3^+} f(x) =$ ______________________________

b. Test your conjecture by evaluating the limit on the Home Screen. Was your conjecture correct? ____________________

5. Make and test a conjecture: As $x \to a$, $f(x) \to$?

a. Make a conjecture of the value of the limit from both directions. $\lim_{x \to 3} f(x) =$ ______________________

b. Test your conjecture by evaluating the limit on the Home Screen. Was your conjecture correct? ______________

c. Evaluate $f(3)$ = ____

B. Examples to Investigate

In this section you are given three more examples to investigate, using the same method that you used in Part A. It will save you a lot of keystrokes if you take the time now to create a **Text Script** of the commands that you entered on the Home Screen.

From the Home Screen, press F1: Tools. Then press 2: Save Copy As... In the next screen, move down to assign the variable name **limita**. When you press **ENTER** the commands on the Home Screen are saved as a Text File.

Detailed instructions for creating a Text Script were given in Chapter 1, Lesson 3. Whenever you want to explore the limit of a function as x approaches a, you can use this script to investigate the limit numerically.

TECH-TIPS **Using a Saved Text Script**

- Press [APPS] [8] to open the Text Editor.
- Select the appropriate options to open the file you created using the commands in this Example.
- Press [F3] [1] for Script view.
- Press [F4] to execute each command.
- Redefine y1(x) as required
- Store the new value for $x \to a$.

For each example below, first graph the function. Then use the text script that you have just created to find the value of the expressions in (a) through (d). After you complete each example, press [◆] [⏶] to move back to the top of the **Text Script**.

(a) $\lim_{x \to a^-} f(x)$ (b) $\lim_{x \to a^+} f(x)$ (c) $\lim_{x \to a} f(x)$ (d) $f(a)$

6. Let $f(x) = \dfrac{(x-3)\cdot(x-2)}{(x-3)}$, $a =$ [illegible]5

a. __________ **b.** __________ **c.** __________ **d.** __________

7. Let $f(x) = \dfrac{\sin x}{x}$, $a = 0$

a. __________ **b.** __________ **c.** __________ **d.** __________

8. Let $f(x) = \dfrac{\cos(x) - 1}{x}$, $a = 0$

a. __________ **b.** __________ **c.** __________ **d.** __________

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C. Test Your Understanding

Now that you have experienced limits of different types of functions, you need to make some generalizations. Use the information you have recorded in Sections A and B to help you draw conclusions and answer these questions.

9. In each case, as you filled out the table, how did the *x*-values change? ______________________________

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10. In each case, as you filled out the table, how did the function values, *f*(*x*), change? ______________________________

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11. Do you think that *f*(*a*) must equal the limit of *f*(*x*) as $x \to a$? Why or why not? ______________________________

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12. Does *f*(*x*) have to be defined at $x = a$ in order for the limit to exist? ______________________________

____

13. Does the left-handed limit have to equal the right-handed limit in order for the limit to exist? ______________________________

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14. Use graphs and the **Text Script** you have created to explore the function $f(x) = [x]$. Use catalog floor(x) to define the function. Does the limit of *f*(*x*) as $x \to 3$ exist? Why or why not?

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Journal Entries

Write the following statements in your journal and indicate whether each is True or False. If false, give a counter example and sketch the graph.

1. If a function has a limit, *L*, as *x* approaches *c*, then it must also be true that:

a. $\lim_{x\to c^+} f(x) = L$

b. $\lim_{x\to c} f(x) = f(c)$

c. $\lim_{x\to c^-} f(x)$ exists and $\lim_{x\to c^+} f(x)$ exists

d. $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$

e. $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$

f. $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$ and $f(c)$ exists

> **NOTE**
> If *f*(*x*) is not defined to the left of $x = c$, then *f* does not have a left-hand limit at *c*.
> If *f*(*x*) is not defined to the right of $x = c$, then *f* does not have a right-hand limit at *c*.

2. Which of the equations above is the best way to say that $\lim_{x\to c} f(x)$ exists? Explain.