

Introduction

The authors, Larry Peterson and Brenda Batten, are high school AP Calculus teachers who have been closely involved with the AP Calculus Exam, both as readers and as consultants. The questions contained in this MathBox have been developed to address the new AP exams (given since May 1998) requiring a graphing calculator.

An outline of AP Calculus topics is included for reference below. Please note that there are worksheets with varying numbers of questions for nearly all of the objectives. Some objectives from topic IV. Polynomial Approximation & Series are not directly addressed.

AP Calculus Topic Outline

Note: An asterisk indicates that the topic or objective is part of the Calculus BC curriculum.

I. Functions, Graphs, and Limits

- A. Analysis of Graphs
- B. Limits of functions
 - 1. Algebraic techniques
 - 2. Estimating limits from tables and graphs
- C. Asymptotic behavior
 - 1. Graphical behavior
 - 2. Describing asymptotic behavior with limits involving infinity
 - 3. Relative magnitudes of functions and their rates of change
- D. Continuity of functions
 - 1. Continuity in terms of limits
 - 2. Geometric interpretations of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)
- * E. Parametric, polar, and vector functions

II. Derivatives

- A. Concept of the derivative.
 - 1. Defined as the limit of difference quotient
 - 2. Relationship between differentiability and continuity
- B. Derivative at a point
 - 1. Slope of a curve at a point - Vertical tangents and no tangents
 - 2. Tangent lines and local linear approximation
 - 3. Instantaneous rate of change as the limit of average rate of change
 - 4. Approximate rate of change from tables and graphs
- C. Derivative as a function
 - 1. Corresponding characteristics of the graphs of f and f'
 - 2. Relationship between incr/dec behavior of f and sign of f'
 - 3. Mean Value Theorem and its geometric consequences
 - 4. Equations involving derivatives - Verbal descriptions

AP Calculus

Topic Outline

- D. Second Derivatives
 - 1. Relationships between the graphs of f , f' , and f''
 - 2. Relationship between sign of f'' and concavity of f
 - 3. Points of inflection
- E. Applications of derivatives
 - 1. Analysis of curves - monotonicity and concavity
 - 2. Optimization - absolute and relative extrema
 - 3. Modeling rates of change - related rates
 - 4. Implicit differentiation - derivatives of inverse functions
 - 5. Rates of change in contextual examples
 - * 6. Analysis of curves in parametric, polar, and vector format
 - * 7. Slope fields and derivatives of implicitly defined functions
 - * 8. Euler's method as numerical solution of differential equations
 - * 9. L'Hopital's Rule and convergence of improper integrals and series
- F. Computation of derivatives
 - 1. Basic formulas
 - 2. Sum, product, and quotient rules
 - 3. Chain rule
 - 4. Implicit differentiation
 - * 5. Derivatives of parametric, polar, and vector functions

III. Integrals

- A. Riemann sums
 - 1. Riemann sums with equal subdivisions
 - 2. Computations with left, right, and midpoint evaluations
- B. Interpretations and properties of definite integrals
 - 1. Definite integrals as a limit of Riemann sum
 - 2. Def. integral of rate of change of quantity, as change over an interval
 - 3. Basic properties of definite integrals
- C. Applications of integrals - int. of rate of change gives accumulated change
 - 1. Area
 - 2. Volumes of solids of revolution
 - 3. Volumes of solids of known cross sections
 - 4. Average value of functions
 - 5. Particle motion
 - * 6. Length of a curve - parametric form
 - * 7. Integrals of vector expressions
- D. Fundamental Theorem of Calculus
 - 1. Evaluations of definite integrals
 - 2. Analysis of anti-derivatives and functions defined as integrals
- E. Techniques of integration
 - 1. Basic integrals
 - 2. Change of variables - change of limits
 - * 3. Integration by parts
 - * 4. Partial fractions
 - * 5. Improper integrals as limits of definite integrals

AP Calculus

Topic Outline

- F. Applications of anti-derivatives
 - 1. Particle motion - initial value problems
 - 2. Separable differential equations
 - * 3. Logistic models
- G. Numerical approximations
 - 1. Riemann sums
 - 2. Trapezoidal rule
 - 3. Tables and graphs

***IV. Polynomial Approximations & Series**

- * A. Concept of series
 - * 1. Sequence of partial sums
 - * 2. Convergence & divergence
- * B. Series of constants
 - * 1. Decimal expansion
 - * 2. Geometric series with applications
 - * 3. Harmonic series
 - * 4. Alternating series with error bound
 - * 5. Area approximations - integral test - convergence of p -series
 - * 6. Ratio test for convergence and divergence
 - * 7. Comparing series for convergence or divergence
- * C. Taylor Series
 - * 1. Taylor polynomial approximation with graphical demonstration of convergence
 - * 2. General Taylor Series about $x = a$
 - * 3. Basic Maclaurin series
 - * 4. Derivatives and anti-derivatives of Taylor series
 - * 5. Formation of new series from old series
 - * 6. Functions defined by power series and radius of convergence
 - * 7. Lagrange error bound for Taylor polynomials

I. B. 2. Estimating limits from tables and graphs

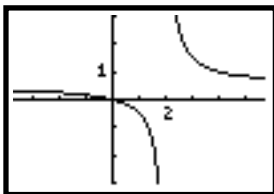
1. Given $f(x) = \sin\left(\frac{1}{x^2}\right)$. Find $\lim_{x \rightarrow 0} f(x)$.

- a) -0.350
- b) -0.306
- c) -0.506
- d) 0.888
- e) does not exist

2. Given $f(x) = x \cdot \sin\left(\frac{1}{x}\right)$. Find $\lim_{x \rightarrow 0} f(x)$.

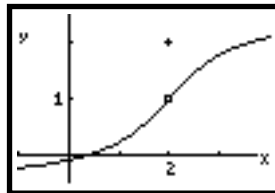
- a) 0
- b) 1
- c) $+\infty$
- d) $-\infty$
- e) does not exist

3. Let $f(x)$ be the function whose graph is shown below:



- a) $\lim_{x \rightarrow 2} f(x) = \infty$
- b) $\lim_{x \rightarrow 2^-} f(x) = -\infty$
- c) $\lim_{x \rightarrow 2^+} f(x) = 3$
- d) $\lim_{x \rightarrow 2^-} f(x) = -3$
- e) none of the above

4. A graph of the function $f(x)$ is given below:



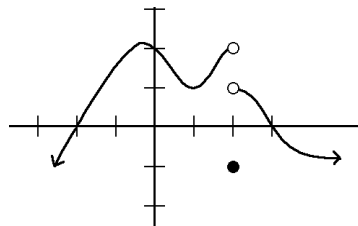
Which of the following statements must be true?

- I. $\lim_{x \rightarrow 2^-} f(x) = 1$
- II. $\lim_{x \rightarrow 2^+} f(x) = 2$
- III. $\lim_{x \rightarrow 2} f(x)$ does not exist

- a) I only
- b) II only
- c) III only
- d) I and II only
- e) I and III only

5. The graph of $f(x)$ is shown in the figure.

What is $\lim_{x \rightarrow 2} f(x)$?



- a) -1
- b) 1
- c) 2
- d) it varies
- e) does not exist

I. D. 2. Geometric interpretations of graphs of continuous functions

(Intermediate Value Theorem and Extreme Value Theorem)

1. Let $f(x)$ be a continuous function on the interval $-2 \leq x \leq 2$. Use the table of values below to determine which of the following statements is true.

x	-2	-1	0	1	2
$f(x)$	-4	1	2	3	-5

- $f(x)$ is differentiable on the interval $[-2, 2]$.
 - The absolute minimum value of $f(x)$ is -5 .
 - $f(x)$ has an absolute maximum value on the interval $[-3, 3]$.
 - A zero of $f(x)$ is 2 .
 - There are at least 2 zeroes of $f(x)$.
2. Let $f(x)$ be a continuous function on the interval $-2 \leq x \leq 2$. Use the table of values below to determine which of the following statements must be true.

x	-2	-1	0	1	2
$f(x)$	-4	1	6	3	-5

- $f(x)$ takes on the value of 5
 - A zero of $f(x)$ is between -2 and -1
 - A zero of $f(x)$ is 6
- I only
 - II only
 - III only
 - I and II only
 - I, II, and III
3. Let $f(x)$ be a continuous function on the interval $-2 \leq x \leq 2$. Use the table of values below to determine which of the following statements must be true.

x	-2	-1	0	1	2
$f(x)$	-4	1	6	3	-5

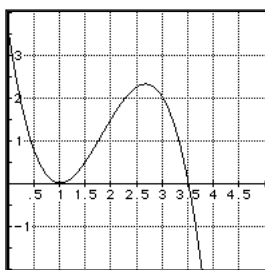
- For some value of x on the interval $[-2, 2]$, $f'(x) = 0$.
 - $f(x)$ has an absolute maximum value on the interval $[-2, 2]$.
 - $f(x)$ takes on the value of 5 on the interval $[-2, 2]$.
- I only
 - II only
 - III only
 - I and II only
 - I, II, and III

II. C. 1. Corresponding characteristics of the graphs of f and f'

1. Given $f'(x) = \sin(x^2)$. The graph of $f(x)$ is increasing on which of the following intervals:

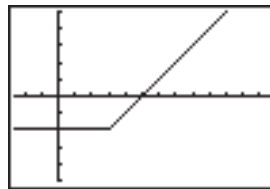
- $(0, \pi)$
- $(\pi, 2\pi)$
- $(-\sqrt{\pi}, \sqrt{\pi})$
- $(\sqrt{\pi}, \sqrt{2\pi})$
- $(-\sqrt{\pi}, \sqrt{\pi})$

2. The graph of $f'(x)$ is shown in the figure. Which of the following must be true about the graph of f at $x = 3$?



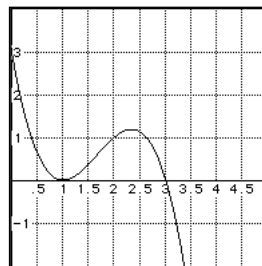
- f is decreasing and concave down
- f is decreasing and concave up
- f is increasing and concave down
- f is increasing and concave up
- f is positive

3. The graph of f' is shown below. Based on the graph, which of the following statements is true?



- f is discontinuous at $x = 3$
- f is increasing for $3 < x < 7$
- f has a local maximum at $x = 5$
- f has a local minimum at $x = 5$
- f is constant for $0 < x < 3$

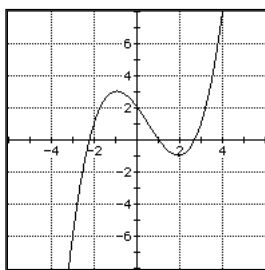
4. The graph of $g'(x)$ is shown in the figure.



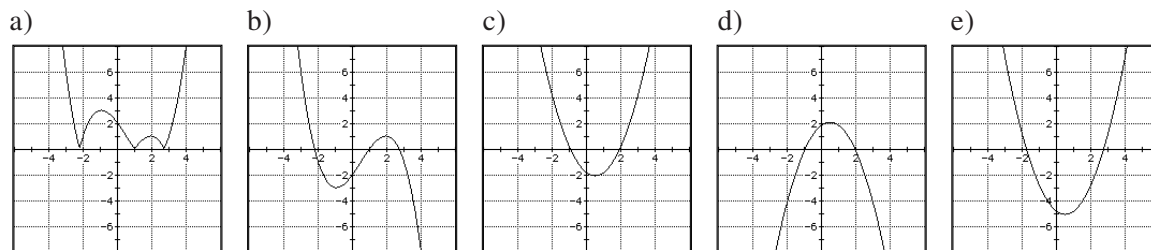
Which of the following has the largest value?

- $g(0)$
- $g(1)$
- $g(2)$
- $g(3)$
- $g(4)$

5. The graph of $f(x)$ is shown.



Which of the following could be the graph of the derivative of $f(x)$?



III. C. 4. Average value of functions

1. Given a graph of a continuous, differentiable function, $f(x)$, such that $f(a) = c$ and $f(b) = d$. Geometrically, the average value of $f(x)$ over the interval $a \leq x \leq b$ is represented by:

- the length of a horizontal segment
- the length of a vertical segment
- an area
- the slope of the line segment with endpoints (a, c) and (b, d)
- the slope of the tangent line at $x = e$, for some value e , where $a \leq e \leq b$

2. Given a function $f(x)$ for which $\frac{dy}{dx} = e^{\sin(x)}$.

The average rate of change of $f(x)$ on the

interval $\left[\frac{-\pi}{2}, \pi\right]$ is:

- 1.506
 - 7.081
 - 0.632
 - 2.978
 - 1.502
3. At what value of x does $f(x) = x e^{2x}$ take on its average value on the interval $[-2, 1]$?

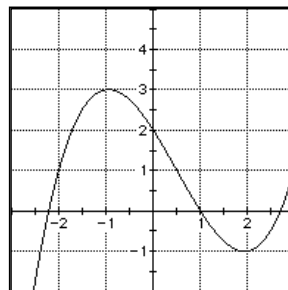
- 0.212
- 0.325
- 0.623
- 1.870
- 5.610

4. If $f(c) = \frac{\int_0^2 f(x) dx}{2-0}$, where $f(x) = e^x - x^2$,

then $c =$

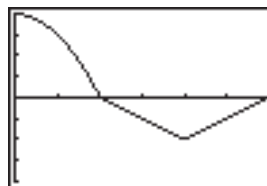
- 0.703
- 1.179
- 1.861
- 2.091
- 3.722

5. Let $H(x) = \int_{-2}^x f(t) dt$ for $-2 \leq x \leq 3$, where $f(t)$ is the function shown in the figure.



For what value of x does $H(x)$ reach a local maximum?

- 2
 - 1
 - 0
 - 1
 - 2
6. The graph of $v(t)$ is shown for $0 \leq t \leq 6$.



$$y = \begin{cases} 4 - t^2, & 0 \leq t \leq 2 \\ |t - 4| - 2, & 2 < t \leq 6 \end{cases}$$

What is the average acceleration from $t = 2$ to $t = 5$?

- $-\frac{7}{2}$
- $-\frac{1}{3}$
- 1
- $\frac{7}{2}$
- cannot be determined

AP Calculus

Name _____

Date _____

III. E. 3. Integration by parts

1. Evaluate $\int x^2 \ln(x) dx$.

a) $x^2 \ln(x) - x^2 + C$

b) $x^3 \ln(x) - x^3 + C$

c) $\frac{x^3 \ln(x)}{3} - \frac{x^3}{3} + C$

d) $\frac{x^3 \ln(x)}{3} - \frac{x^3}{6} + C$

e) $\frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C$

2. Evaluate $\int e^x \sin(x) dx$.

a) $e^x \sin(x) + C$

b) $e^x \sin(x) - \cos(x) + C$

c) $e^x \sin(x) - e^x \cos(x) + C$

d) $\frac{1}{2} e^x \sin(x) - \frac{1}{2} e^x \cos(x) + C$

e) $\frac{1}{2} e^x \sin(x) - \frac{1}{4} e^x \cos(x) + C$

3. Evaluate $\int x^2 e^{3x} dx$.

a) $(x^2 - x)e^{3x} + C$

b) $(3x^2 - 6x + 6)e^{3x} + C$

c) $\left(x^2 - \frac{2x}{3} + \frac{2}{27}\right)e^{3x} + C$

d) $\left(\frac{x^2}{3} + \frac{2x}{9} + \frac{2}{27}\right)e^{3x} + C$

e) $\left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27}\right)e^{3x} + C$

4. $\int x^3 \ln x dx =$

a) $x^4 \ln x - \frac{x^4}{4} + C$

b) $3x + C$

c) $x^3 \ln x - \frac{x^3}{3} + C$

d) $\frac{x^3}{4} \ln x + C$

e) $x^4 \ln x - \frac{x^4}{16} + C$

5. $\int_e^{2e} \ln x dx =$

a) 1

b) e

c) $2e$

d) $4e$

e) $\frac{3e^2}{2}$

6. $\int x \cos 2x dx =$

a) $\frac{x}{2} \sin 2x - \frac{1}{2} \cos 2x + C$

b) $\frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x + C$

c) $-\frac{x}{2} \sin 2x - \frac{1}{4} \cos 2x + C$

d) $2x \sin 2x - \frac{1}{2} \sin 2x + C$

e) $2x \sin 2x + 4 \cos 2x + C$