T1. Let $\underline{A}\,\underline{B}\,\underline{C}$ represent a three-digit base 10 number whose digits are A, B, and C with $A \geq 1$.
Compute the minimum value of $\underline{A}\,\underline{B}\,\underline{C} - (A^2 + B^2 + C^2)$.

T2. In ΔABC, side BC is the average of the other two sides. If $\cos \angle C = \frac{AB}{AC}$ compute the numerical value of $\cos \angle C$.

T3. Compute the number of distinct ways in which 77 one-dollar bills can be distributed to 7 people so that no person receives less than \$10.

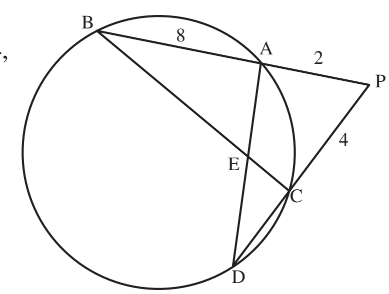
T4. Starting at the same time at corner M, Alison and Ben run in opposite directions around square track MNPQ, each travelling at a constant rate. The first time they pass each other at corner P is the tenth time that they meet. Compute the smallest possible ratio of the faster person's speed to the slower person's.

T5. Determine all integer values of θ with $0° \leq \theta \leq 90°$ for which $(\cos\theta + i\sin\theta)^{75}$ is a real number.

T6. Assuming the expression converges, determine the largest integer n with $n \leq 4{,}000{,}000$ for which
$\sqrt{n+\sqrt{n+\sqrt{n+\ldots}}}$ is rational.

T7. A trapezoid has a height of 10, its legs are integers, and the sum of the sines of the acute base angles is $\frac{1}{2}$.
Compute the largest sum of the lengths of the two legs.

T8. Points A, B, C, and D lie on the given circle. If AB = 8, AP = 2, and PC = 4, determine the ratio of the area of quadrilateral PAEC to the area of ΔBAE.



T9. Points C and D lie on opposite sides of line $\overline{AB}$. Let M and N be the centroids of ΔABC and ΔABD respectively. If AB = 25, BC = 24, AC = 7, AD = 20, and BD = 15, compute MN.

T10. If $\log_{10} 14 = x$, $\log_{10} 15 = y$ and $\log_{10} 16 = z$, then determine the number of elements in $S = \{\log_{10} 1, \log_{10} 2, \log_{10} 3, \ldots, \log_{10} 100\}$ which can be written in the form $ax + by + cz + d$ for rational numbers a, b, c, and d.

The power question is worth 40 points. Each of the ten parts is worth 4 points. To receive full credit the presentation must be legible, orderly, clear, and concise. The pages submitted for credit should be NUMBERED IN CONSECUTIVE ORDER AT THE TOP OF EACH PAGE in what your team considers to be proper sequential order. PLEASE WRITE ON ONE SIDE OF THE ANSWER PAPERS ONLY.

Put the **TEAM NUMBER** (not the team name) on the cover sheet used as the first page of the papers submitted. Do not identify the team in any other way.

Consider a collection of piles of bananas consisting of one pile of 9 bananas, one pile of 6 and one pile of 2. Such a collection, C_1, could be expressed as (9, 6, 2). Obtain a new collection, C_2, by harvesting C_1 where harvesting is defined to mean remove one banana from each pile to form a new pile. Thus C_2 = (8, 5, 3, 1) and, if C_2 is harvested, we obtain C_3 = (7, 4, 4, 2).

1. **a.** Let C_1 = (8, 5, 2). Determine C_2, C_3, C_4, and C_{100}.
b. Given the collection M_1 = (7, 6, 5), determine M_{1995}. Explain how you arrived at that result.

2. Prove that the collection (k, k − 1, . . . , 3, 2, 1) remains fixed after harvesting.

3. Prove that any collection of piles whose total number of bananas is 6 can be reduced to the collection (3, 2, 1) by successive harvesting. Determine the maximum number of harvests required.

4. Denote by $C(n_k, \ldots, n_1)$ a collection of k piles of size n_k through n_1 ordered such that $n_k \geq n_{k-1} \geq \ldots \geq n_1$. Suppose $n_k + n_{k-1} + \ldots + n_1 = 7$. Determine the number of collections C such that the 1995th harvest yields the collection (4, 2, 1). Justify your answer.

5. Let C = (n), that is, let collection C be one pile of n bananas. If C is harvested successively, the collections will eventually fall into a periodic sequence. That is, for all k sufficiently large $C_k = C_{k+p}$. If the harvest is fixed after some point then p = 1. Determine all values of n such that p = 1995. Explain your reasoning; you need not prove your result.

I-1. Compute the largest prime factor of:

$$3(\,3(\,3(\,3(\,3(\,3(\,3(\,3(\,3(\,3 + 1) + 1) + 1) + 1) + 1) + 1) + 1) + 1) + 1) + 1) + 1$$

I-2. Triangle ABC is similar to ΔMNP. If $BC = 60$, $AB = 12$, $MP = 8$, and $BC > AC > AB$, compute the sum of all possible integer values for the ratio of the area of ΔABC to the area of ΔMNP.

I-3. Determine all positive primes p such that $p^{1994} + p^{1995}$ is a perfect square.

I-4. Let $.\overline{A} = .AAAA\ldots$ Compute the number of distinct ordered triples (A, B, C) with $A, B, C \in \{0, 1, 2, \ldots 9\}$ such that k is an integer if

$$k = \frac{.\overline{ABC} + .\overline{ACB} + .\overline{BAC} + .\overline{BCA} + .\overline{CAB} + .\overline{CBA}}{.\overline{A} + .\overline{B} + .\overline{C}}$$

I-5. Compute the number of distinct planes passing through at least three vertices of a given cube.

I-6. Determine all positive integers $k \le 2000$ for which $x^4 + k$ can be factored into two distinct trinomial factors with integer coefficients.

I-7. Let $f(x) = \sqrt{x + 2} + c$. Determine all real values of c such that the graphs of f(x) and its inverse $f^{-1}(x)$ intersect in two distinct points.

I-8. For x and y in radians, compute the number of solutions in ordered pairs (x, y) to the following system:

$$\sin(x + y) = \cos(x - y)$$

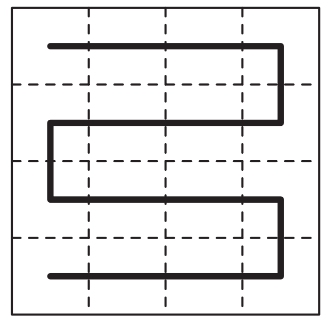
$$x^2 + y^2 = \left(\frac{1995\pi}{4}\right)^2$$

RELAY #1

R1-1. Compute the number of distinct 11 letter palindromic permutations of the letters of MISSISSIPPI.

R1-2. Let K = TNYWR. If the regular octagon ABCDEFGH has sides equal to $\frac{K}{30}$ in length, compute the value of $AC^2 - AD$.

R1-3. Let M = TNYWR. Let S_n be a unit square subdivided into 2^{2n} congruent squares. The centers are all connected as shown in the diagram at the right for S_2. Compute the least value of n such that the sum of the lengths of the connecting line segments exceeds 10^{4M}.



RELAY #2

R2-1. Let ABCDEF be a regular hexagon. If the ratio of the area of region ABCE to the area of the hexagon in simplest terms is $\frac{a}{b}$, compute a + b.

R2-2. Let K = TNYWR. Compute the number of sets of K consecutive integers in {1, 2, 3, . . . , 50} such that the product of the elements of each set of K integers is divisible by both 12 and 21.

R2-3. Let M = TNYWR. Given points D(0, M), O(0, 0), and E(M, 0), equilateral triangle ABC is inscribed in ΔDOE such that C is the midpoint of $\overline{DE}$ and $\overline{AB}$ is parallel to $\overline{DE}$. The length of $\overline{AB}$ can be expressed in simplest form as $a(\sqrt{b} - \sqrt{c})$. Determine (a, b, c).