Name:

Techniques of Integration: *Integration by Substitution*

A World of Inverses

In elementary school, most students remember long division as a dreary and difficult task. One contributing factor, no pun intended, is that in order to be successful at division, students must first conquer multiplication. The secret to being able to divide is in being able to recognize the corresponding “multipliers” or factors.

In Algebra, the secret to solving an equation is in being able to apply the correct inverse operation at the proper time. In Calculus, the secret to solving differential equations is essentially the same. You must first master differentiation. According to the Fundamental Theorem of Calculus, the inverse “operation” that should be applied is integration. Then you must be able to recognize which form of differentiation created the pattern that appears in the integrand.

In this chapter, we assume that you already know the basic forms of antidifferentiation. Use the Home Screen of the TI-89 to assist you when necessary.

Integration by Substitution

I once heard Dr. Frank Demana say that substitution is one of the most profound rules in all of mathematics. In integration, it can tame many a savage beast. We begin with a composite function, find its derivative, and then “undo” the differentiation by integrating.

1. In order to find the derivative of a composite function, you must apply the ____.

2. Let $y = (\sin(x))^3$.

a. The derivative of $y = (\sin(x))^3$ is ____.

b. Write the corresponding integral: ____ $= (\sin(x))^3 + C$

c. What u-substitution could simplify the form of the integrand in #2b?

Let $u =$ ________________.

3. Let $y = \sin(x^3)$.

a. The derivative of $y = \sin(x^3)$ is ____.

b. Write the corresponding integral: ____ $= \sin(x^3) + C$

c. What u-substitution could simplify the form of the integrand in #3b?

Let $u =$ ________________

4. Let $y = f(g(x))$.

a. Explain how you decide which part of the composite function $y = f(g(x))$, should be represented by u when making a u-substitution.

____

b. What must also appear as a part of the integrand?

____

Techniques of Integration: *Integration by Substitution*

5. Verify your answers to #2a, #2b, #3a, and #3b on the Home Screen of your calculator. Write an equation in each space below.

#2a ____________________ #2b ____________________

#3a ____________________ #3b ____________________

a. What theorem do these examples illustrate?

____

b. Write the part of the Fundamental Theorem of Calculus that applies in this case.

____

____

____

____

____

c. State the meaning of this theorem in your own words.

____

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6. Explain how to use substitution as a method of integration.

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Techniques of Integration: *Integration by Parts*

In Lesson 31 you worked with integrands that were composite functions. In this lesson, you will work with integrands that are products of functions. You will also learn to use a **Flash Application** of the TI-89 called the **Symbolic Math Guide**. It can assist you in practicing different techniques of integration.

A. Integrands that are Products of Functions

1. $y = x^2 \cdot ln(x)$

a. Find the derivative. $\frac{dy}{dx} =$ ____________________

b. Write the corresponding integral: ____________________

Verify with the TI-89 that the two functions satisfy the Fundamental Theorem of Calculus.

c. What rule did you use to differentiate #1a? ____________________

d. Complete the equation: $x^2 \cdot ln(x) =$ ____________________ + ____________________

e. Solve the equation in #1d for $\int 2 \cdot x \cdot ln(x)dx$ = ____________________

f. Simplify your expression using pencil and paper. Verify that your answer is equivalent to the one obtained by using the Home Screen of the TI-89. Show your work.

$\int 2 \cdot x \cdot ln(x)dx$ = ____________________

g. The integrand of $\int 2 \cdot x \cdot ln(x)dx$ is the ____________ of two functions, ____________ and ____________.

2. Let $y = u \cdot v$, where u and v are functions of x.

a. Find the derivative. $\frac{dy}{dx} = \frac{d}{dx}(u \cdot v) =$ ____________________

b. Integrate each side of the equation in #2a and simplify in the space below.

c. Solve the resulting equation in #2b for $\int u \cdot dv$ = ____________________

d. In #1e, the integral $\int 2 \cdot x \cdot ln(x)dx$ is of the form $\int u \cdot dv$. Identify the following:

$u =$ ____________________ $dv =$ ____________________

$du =$ ____________________ $v =$ ____________________

3. How do you select which part of the integrand to use for u and which part to use for dv?

____

____

Techniques of Integration: *Integration by Parts*

When selecting $u = f(x)$, look for functions in the following order: logarithmic functions, inverse trig functions, power functions, exponential functions, and then trigonometric functions. You can remember this order by the acrostic LIPET.

4. Refer to #1e and #1f. Why is it helpful to write $\int u \cdot dv$ as it is written in #2c?

____

5. Exploration: Use the Home Screen to evaluate the following integrals; then identify u and dv.

a. $\int \left[x^3 \cdot \ln(x)\right] dx =$ ________ $u =$ ________ $dv =$ ________

b. $\int \left[x^2 \cdot \ln(x)\right] dx =$ ________ $u =$ ________ $dv =$ ________

c. $\int \left[x^1 \cdot \ln(x)\right] dx =$ ________ $u =$ ________ $dv =$ ________

d. $\int \left[x^0 \cdot \ln(x)\right] dx =$ ________ $u =$ ________ $dv =$ ________

e. $\int \ln(t)\, dt =$ ________

B. Symbolic Math Guide

In solving integrals where the integrands are products of functions, the formula that you have just derived can be very useful. This method is called **Integration by Parts.**

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

The difficulty lies in deciding which factor of the integrand should represent u and which should represent dv. At first you will learn by trial and error. You have already learned that the goal is to arrive at an expression for $\int v \cdot du$ that is easier to integrate than the one you started with, $\int u \cdot dv$.

Choose an expression for dv that is easily integrated in order to find the expression v. When selecting u, often there is a function, du that simplifies the integrand on the right-hand side. You have already seen that the function $ln(x)$ is a good choice for u.

Next we will use an Application of the TI-89 called the **Symbolic Math Guide**. This is a free Flash Application which you can download from the TI WEB site. For instructions, see the TECH-TIPS on page 224.

TECH-TIP APPS: **Symbolic Math Guide: TI WEB Site** *page 224*

TECH-TIP APPS: **Symbolic Math Guide: Installation** *page 224*

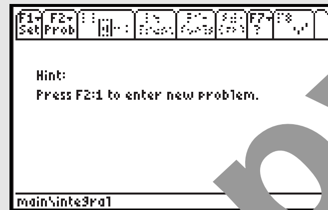
Techniques of Integration: *Integration by Parts*

Learning Techniques of Integration with the Symbolic Math Guide

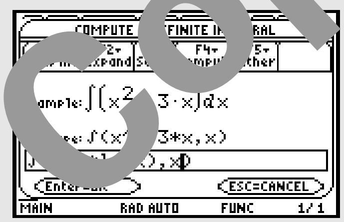
TECH-TIP	[APPS]: Symbolic Math Guide: Using Symbolic Math Guide	*page 225*

[APPS] Flash Applications: **Symbolic Math Guide**

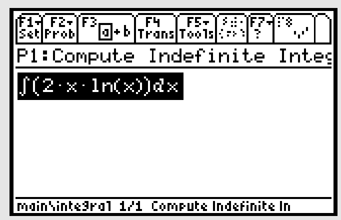
- Create a New Symbolic Math Problem Set
- Name the Variable: **integral**
- [F2], which is Problem, [1] New Problem.



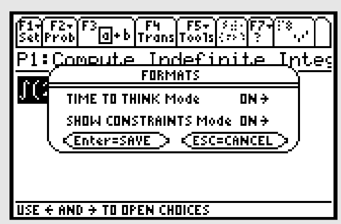
- [F4] Compute, [2] Indefinite Integral.
- Enter $\int 2 \cdot x \cdot ln(x)dx$ using the syntax shown in the screen.



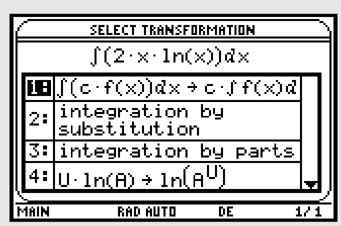
- Press [ENTER] to move to the Problem Screen.
- The bottom line indicates that this is 1/1 in the current Problem Set **integral**.
- The top line indicates that this is problem number **P1**.



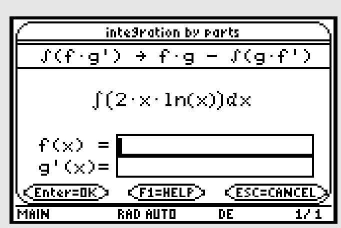
- [F1] Set, [9] Formats.
- **Time to Think** Mode ON.
- **Show Constraints** Mode ON.
- Press [ENTER] to Save.



- [F4] Transformations.



- [3] Integration by Parts.
  Remember that your selection for *dv* or *g´(x)* must be easy to integrate. Do you recognize any part of the integrand as being a derivative of some other function?
- Enter the expression for *f(x)* and *g´(x)* in the text box for each.
- Use the cursor pad, ⏶ ⏷, to move between the text boxes.



Name:

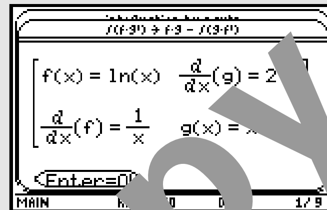
Techniques of Integration: *Integration by Parts*

1. Your selection for u or $f(x)$ should be simpler after you take the derivative.
Your selection for v' or $g'(x)$ should be simpler after you integrate.

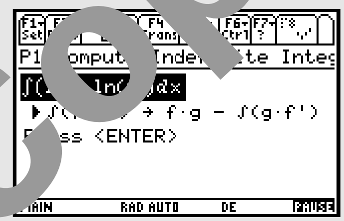
a. $f(x) =$ ______________ **b.** $g'(x) =$ ______________

c. $f'(x) =$ ______________ **d.** $g(x) =$ ______________

- Press [ENTER] to Save your selections and move to the next screen to verify your answers.
- Write the way you think the rule will be applied after you select each transformation.

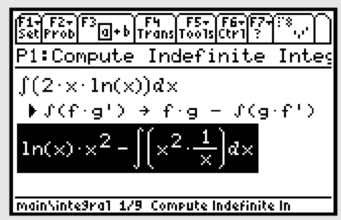


- Press [ENTER] to see the rule displayed for Integration by Parts.
- Since you are in **Time to Think** Mode, the results will not be displayed until you press [ENTER] a second time.
- Before you do that, you should write your own conjecture.

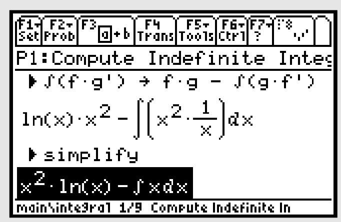


2. **a.** Write your conjecture ____

- [ENTER] The result will be displayed on the screen.
- Verify that your conjecture was correct.



- [ENTER] a second time.
- The algebraic expression in the integrand is automatically simplified.



Technology Etiquette

At this point in your life, you are focusing on techniques of integration. It is proper that the technology at hand assist you with a lower level skill.

Students who have not mastered Algebra at this point can use the **Symbolic Math Guide** to enter other types of problems. When the level of difficulty of the problem is lowered, the Transformation list is adjusted for that particular level. You are encouraged to practice your skills at every level where there is a need.

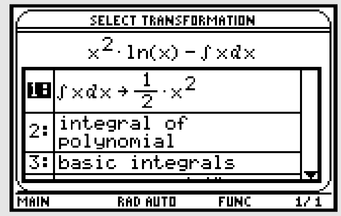
However, when you are learning a new skill, the technology available in the TI-89 can support you by keeping you on the correct path. It is up to you to think through each step as the transformation is applied. It is also up to you to practice enough to become an independent thinker. The technology is your tutor, your guide. Do not let it become your crutch.

TECH-TIP [APPS]: **Symbolic Math Guide: Simplification Rules** *page 227*

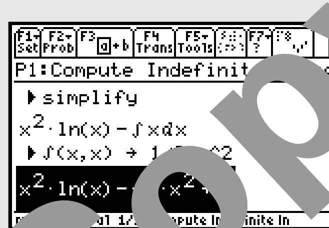
TECH-TIP [APPS]: **Symbolic Math Guide: [F7] Goal** *page 227*

TECH-TIP [APPS]: **Symbolic Math Guide: [F4] Transformations**

- [F4] Transformation. Scroll down to see the list of possible choices.
- Highlight the rule that you prefer and press [ENTER].
- Since you are in **Time to Think** Mode, only the rule is displayed.
- Predict the outcome of applying the rule you have selected.



- Write the predicted result for each step on your paper.
- Beside each step, write the transformation that you used.
- [ENTER] to see the result.



b. Write your final answer in the space below.

____

At any time you may use your cursor keys to move up and down the steps of the problem. Also there are some valuable tools to assist you under [F5] Tools and [2nd] [F2], which is [F7]?.

3. Look under [F5] and [F7] and answer the following questions.

a. Which menu has the Goal of the problem type defined? ______________________

b. What is the domain of the problem in #2? ______________________

c. After selecting [F3] Subexpression Tool, which cursor key do you press to select a smaller subexpression within a problem?
____

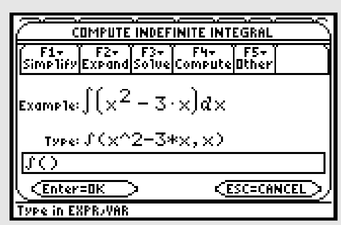
d. What keystrokes do you press to get to "Subexpression Selection Help . . ."? ______________________

e. Where can you download The SMG Guided Tour and the user guide? ______________________

TECH-TIP [APPS]: **Symbolic Math Guide: [F2] Problem: [1] New Problem** *page 228*

Your teacher may give you other problems to practice in small groups. If so, follow the directions below.

- To enter a new problem, press [F2] Problem, [1] New Problem. You are back at the **Compute Indefinite Integral** screen.
- Enter another integral and proceed as outlined in the example above.
- Other options available are: [F1] Simplify, [F2] Expand, [F3] Solve, [F4] Compute, and [F5] Other.
- If you press any of the function keys you will see a sub-menu.
- Press [ESC] to close the menu.



TECH-TIP [APPS]: **Symbolic Math Guide: [QUIT]: Exit SMG** *page 228*

- To exit the Symbolic Math Guide, press [2nd] [ESC], which is [QUIT].

Techniques of Integration: *Integration by Partial Fractions*

The Problem $\displaystyle\int \frac{3x+7}{x^2+5x+6}\,dx$

Small Group Work

Try your hand at integrating the given integral by previous methods. Discuss the problems you encounter. You may use your TI-89 in any way that you choose. If you do so, be ready to show to the class the intermediate steps that justify your answer.

Johann Bernoulli, known as the "Archimedes of his age," suggested a straightforward method of integrating problems like the one above. The method is called **integration by partial fractions**. The process is as elementary as adding two fractions. The difficulty, however, lies in being able to recognize the two fractions that were added to form the resulting integrand.

A. Integration by Partial Fractions

1. Add the following fractions. Fill in the blanks where indicated.

$$\frac{1}{x+2}+\frac{2}{x+3} = \frac{1}{(x+2)}\cdot\frac{(\qquad)}{(\qquad)} + \frac{2}{(x+3)}\cdot\frac{(\qquad)}{(\qquad)} = \frac{1\cdot(\qquad)+2\cdot(\qquad)}{(x+2)\cdot(x+3)} = \frac{(\qquad)\cdot x+\qquad}{(x+2)\cdot(x+3)} = \frac{\qquad}{(x+2)\cdot(x+3)}$$

2. Separate the result into **partial fractions**.

You could easily integrate the following integrals $\displaystyle\int\left(\frac{a}{x+2}\right)dx$ and $\displaystyle\int\left(\frac{b}{x+3}\right)dx$ if you could determine the values of a and b. Then you would know how to integrate:

$$\int\left(\frac{3x+7}{(x+2)\cdot(x+3)}\right) = \int\left(\frac{a}{x+2}\right)dx + \int\left(\frac{b}{x+3}\right)dx$$

So your task is reduced to finding the values of a and b. Let's pretend we don't know the answer already.

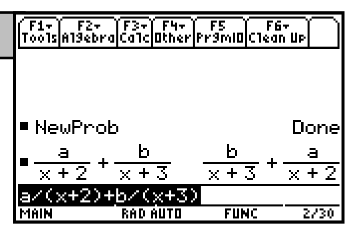
$$\frac{3x+7}{(x+2)\cdot(x+3)} = \frac{a}{(x+2)}+\frac{b}{(x+3)}$$

You know that the denominators of the partial fractions will be $(x + 2)$ and $(x + 3)$. At this point, you do not know what the numerators will be.

- Let a represent the numerator of the first partial fraction.
- Let b represent the numerator of the second partial fraction.
- Use your TI-89 Home Screen.

TECH-TIP [HOME] Screen: [F6] CleanUp: [2] New Problem *page 228*

Begin each section with a **New Problem** command.

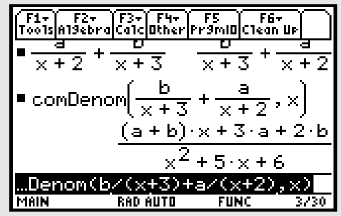


TECH-TIP [HOME] Screen: Entering Fractions *page 228*

Enter the right-hand side of the equation: $\dfrac{a}{(x+2)}+\dfrac{b}{(x+3)}$

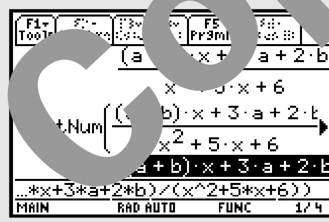
TECH-TIP HOME Screen: F2 Algebra Menu: 6 Common Denominator *page 229*

- Use F2 Algebra Menu: 6 to find the **Common Denominator** of the expression.



TECH-TIP HOME Screen: F2 Algebra Menu: B: Extract Numerator *page 229*

- Use F2 Algebra, ⊙ ⊙: Extract, 1 get Numerator(
- ⊙ ENTER To highlight and paste the previous expression.
-) close parenthesis, ENTER to execute the command.



3. Complete the equations:

a. $\frac{3x + 7}{(x + 2)\cdot(x + 3)} = \frac{a}{(x + 2)} + \frac{b}{(x + 3)} = \frac{a\cdot(\qquad)}{(x + 2)\cdot(\qquad)} + \frac{\qquad(\qquad)}{(x + 3)\cdot(\qquad)}$

b. $\frac{3x + 7}{(x + 2)\cdot(x + 3)} = \frac{\rule{4cm}{0.4pt}}{(\qquad)(\qquad)} = \frac{(\qquad)\cdot x + \rule{1cm}{0.4pt}}{(x + 2)\cdot(x + 3)}$

In order for the expressions in #3b to be equivalent, the numerators must be equivalent.

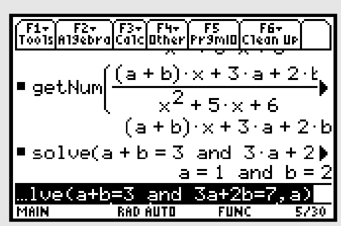
c. $3x + 7 =$ ____

You must now find the solution of a system of two equations in terms of the variables, *a* and *b*.
Write the equations that must be solved in order to determine the values of *a* and *b* that are independent of *x*.

d. $a + b =$ ___________ and $3\cdot a + 2\cdot b =$ ___________

TECH-TIP HOME Screen: F2 Algebra Menu: 1 Solve a System of Equations *page 229*

- Use F2 Algebra, 1 Solve.
- "Solve(a + b = 3 and 3 · a + 2 · b = 7, *a*)"
- Use alpha = for "*a*" and alpha (for "*b*".
- Use CATALOG = ⊙ for the word "and". Then press ENTER.



e. $a =$ ___________ $b =$ ___________

Techniques of Integration: *Integration by Partial Fractions*

4. Substitute
Therefore, the fraction can be separated into the partial fractions below:

$$\frac{3x + 7}{(x + 2) \cdot (x + 3)} = \frac{a}{(x + 2)} + \frac{b}{(x + 3)} = \underline{\qquad\qquad\qquad\qquad}$$

5. $\int \frac{3x + 7}{x^2 + 5x + 6}\,dx$. Separate into partial fractions. Then use the basic rules of integration to complete your answer. Show your work in the space below. Check your answer on the Home Screen.

B. Take another look at the problem

Step 1: The two fractions on the right side of the equation are combined.

The Original Problem: $\frac{3x + 7}{(x + 2) \cdot (x + 3)} = \frac{a}{(x + 2)} + \frac{b}{(x + 3)}$

$$\frac{3x + 7}{(x + 2) \cdot (x + 3)} = \frac{a \cdot (x + 3) + b \cdot (x + 2)}{(x + 2) \cdot (x + 3)}$$

Step 2: In order for these equations to be equivalent, the numerators must be equal.

$3x + 7 = a \cdot (x + 3) + b \cdot (x + 2)$

1. Solve for a.

I. $(3x + 7) - b(x + 2) = a(x + 3)$

II. $\frac{(3x + 7) - b(x + 2)}{(x + 3)} = a$

a. What value of x will eliminate the b-term in Equation I? $x =$ ________________

b. Which **term** in Equation II will be eliminated when you substitute $x = -2$? _______________

c. Re-write Equation II without that term. ________________

d. Substitute $x = -2$. Evaluate the resulting expression for a.

Techniques of Integration: *Integration by Partial Fractions*

2. Solve for b.

a. Solve Equation I for b. Show your work. Label the resulting equation III.

I. $3 \cdot x + 7 = a \cdot (x + 3) + b \cdot (x + 2)$

b. What value of x will eliminate the a-term in Equation I? x = __________________________

c. Which **term** in Equation III will be eliminated when you substitute $x = -3$? ______________

d. Re-write Equation III without that term. ____

e. Substitute $x = -3$. Evaluate the resulting expression for b. ______________________________

Separate the Fraction into Partial Fractions

3. In the space below, re-write the fraction $\frac{3x + 7}{(x + 2) \cdot (x + 3)}$ as the sum of two partial fractions.

4. Observe the Pattern $\frac{3x + 7}{(x + 2) \cdot (x + 3)} = \frac{a}{(x + 2)} + \frac{b}{(x + 3)}$

This algebraic solution suggests a more efficient method. Review the steps you used to solve for a and b by completing the following statements.

a. Write the expression you used to evaluate a and b. a = ____________________ b = ____________________

b. When solving for a, the factor ________ is missing from $\frac{3x + 7}{(x + 2) \cdot (x + 3)}$, the denominator of the original fraction.

The factor that is missing is the _____________ of the original a-term.

c. How did you determine the value to substitute for x when solving for the numerator of the a-term? Write your answer in a complete English sentence.

____

____

d. When solving for b, the factor ________ is missing from $\frac{3x + 7}{(x + 2) \cdot (x + 3)}$, the denominator of the original fraction.

The factor that is missing is the ________________ of the original b-term.

e. How did you determine the value to substitute for x when solving for the numerator of the b-term? Write your answer in a complete English sentence.

____

____

Name:

Techniques of Integration: *Integration by Partial Fractions*

NOTE

The last section has been preparing you to understand a "shortcut method" of doing integration by partial fractions. I present this to you with fear and trembling. My fear is that you will remember the shortcut **only;** and not understand the reasoning behind it.

Therefore, remember this: I present this "shortcut" to you because time is an issue on the AP® Calculus Exam. You must give me your word as ladies and gentlemen that you promise to remember all of the proper Algebra from whence it came. What I am about to tell you is certainly not proper; but it does save a little time.

Shortcut Method: Integration by Partial Fractions

Solve for *a* and *b*

$$\frac{m \cdot x + k}{(x-p)\cdot(x-t)} = \frac{a}{(x-p)} + \frac{b}{(x-t)} \quad \Rightarrow m \cdot x + k = a \cdot (x-t) + b \cdot (x-p)$$

- Since $x = p$ eliminates the term $b(x - p)$ in the equation $m \cdot x + k = a \cdot (x-t) + b \cdot (x-p)$; and the solution for a becomes a familiar form, we can say:

  To find the value for a, cover the factor (x – p) in the denominator of the original fraction: $\dfrac{m \cdot x + k}{\cancel{(x-p)}\cdot(x-t)}$.

  Evaluate $a = \dfrac{m \cdot x + k}{(x-t)}$, when $x = p$.

- Since $x = t$ eliminates the term $a(x\text{-}t)$ in the equation $m \cdot x + k = a \cdot (x-t) + b \cdot (x-p)$; and the solution for b becomes a familiar form, we can say:

  To find the value for b, cover the factor $(x - t)$ in the denominator of the original fraction: $\dfrac{m \cdot x + k}{(x-p)\cdot\cancel{(x-t)}}$.

  Evaluate $b = \dfrac{m \cdot x + k}{(x-p)}$, when $x = t$.

5. Try this one: $\int \left[\tan^{-1}(x) \cdot x\right] dx$

Use **Symbolic Math Guide.**

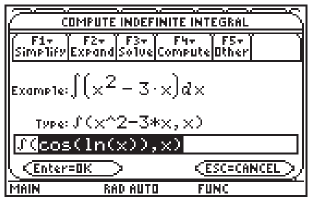
Name:

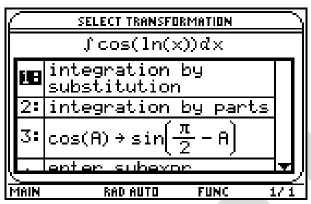
Techniques of Integration: *Symbolic Math Guide Practice & Summary*

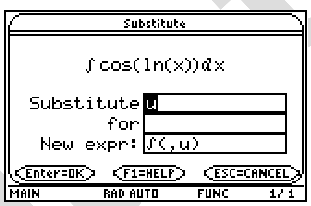
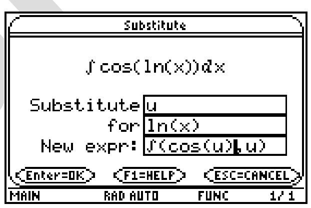
An example is given below that illustrates the use of the **Symbolic Math Guide.**
It also stretches your skills in the techniques of integration.

A. EXAMPLE $y=\int cos\left(ln\left(x\right)\right)dx$

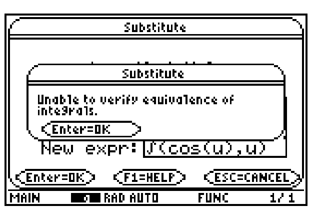
<table>
<tr><th>Follow the instructions in this column.</th><th>The resulting screen</th><th>Use this colum to show your work.
Use proper Calculus notation.</th></tr>
<tr><td>Press [APPS]: Highlight SMG or Symbolic Math Guide.

If you have an older version TI-89, press [APPS] [1] Flash App.</td><td></td><td>Press [F2] [1] to enter a New Problem.
Press [F4] Compute: [2] Indefinite Integral.
Enter the integrand as shown on the screen.

$y=\int cos\left(ln\left(x\right)\right)dx$</td></tr>
<tr><td>Press [ENTER].</td><td></td><td></td></tr>
<tr><td>Press [F4] Transformation.</td><td></td><td></td></tr>
<tr><td>Select [1] integration by substitution.

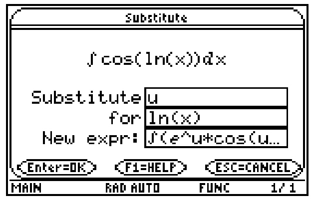
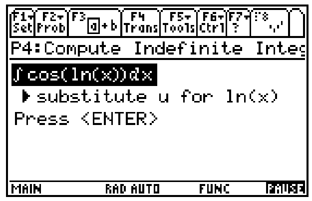
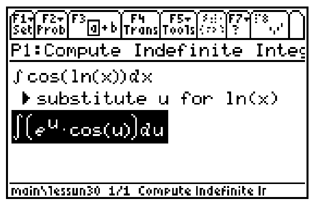
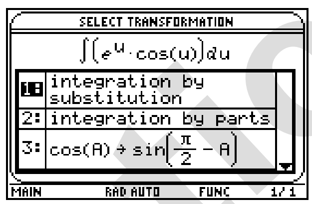
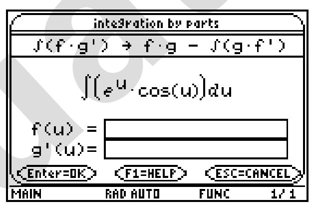
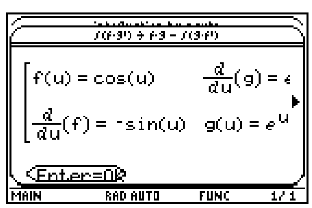
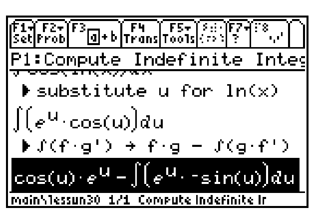
You may use a different variable for substitution other than *u* if you prefer.</td><td></td><td>Write the formula you will apply.</td></tr>
<tr><td>Enter the expression that *u* represents. Then enter the new integrand **as shown at the right.**</td><td></td><td>*u = ln(x)*

$\int cos\left(u\right)du$</td></tr>
<tr><td>Press [ENTER] twice to see if you are correct.

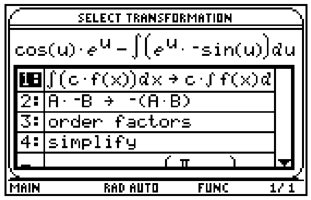
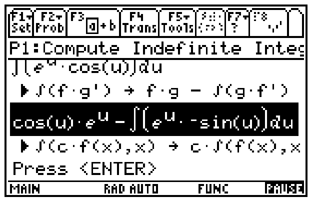
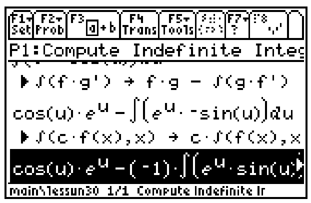
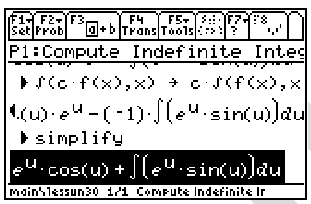
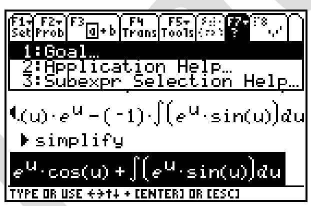
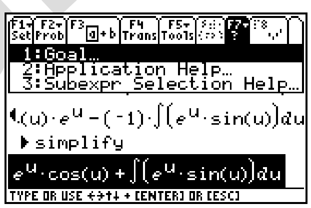
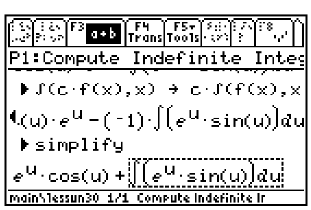
Press [ENTER] to return to the previous screen.</td><td></td><td>Remember that when using integration by substitution, the derivative of *u* must also be part of the integrand.

The message tells you that our choice is probably incorrect.</td></tr>
</table>

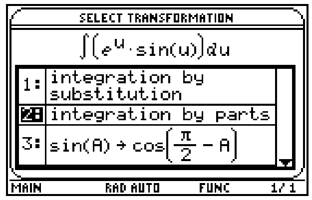
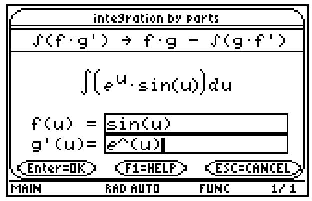
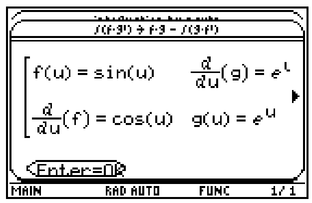
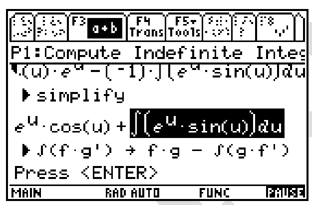
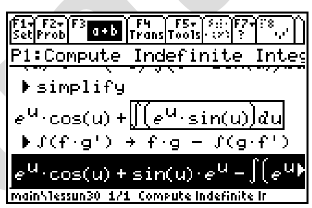
Techniques of Integration: *Symbolic Math Guide Practice & Summary*

Follow the instructions in this column.	The resulting screen	Use this colum to show your work. Use proper Calculus notation.
Press [F1] Help to see the correct substitution. Press ◀ and ▶ to see the entire expression.		Show the intermediate steps for the substitution. *u* = _______________ *du* = _______________
Press [ENTER] twice to return to the main screen. **Time to Think** mode shows you the formula you have selected and gives you the opportunity to write what you expect.		Write the integral that you expect as a result of the substitution.
Press [ENTER] to see if you were correct.		Write the resulting integral here.
Press [F4] Transformation.		Write the formula that you will apply.
Select [2] integration by parts. Enter your choice for *f*(*u*) and *g*´(*u*).		*f(u)* = _______________ *g'(u)* = _______________ *f'(u)* = _______________ *g(u)* = _______________
Press [ENTER] twice to see your results. Press ▶ to scroll to the right of the screen.		Write the resulting equation.
Press [ENTER] to see the formula. Write the equation that you expect. Then press [ENTER] to see the result.		Check to see that your answer agrees.

Techniques of Integration: *Symbolic Math Guide Practice & Summary*

Follow the instructions in this column.	The resulting screen	Use this colum to show your work. Use proper Calculus notation.
Press [F4] Transformation. Select number [1]. Then press [ENTER].		Write the formula that you will apply.
Press [ENTER] to see the formula.		Apply the formula and write the resulting equation.
Press [ENTER] to see the results. Press (▸) to see the entire expression.		Check to see that your answer agrees.
Press [ENTER] to simplify.		Write the simplified equation.
Is this the end of the problem? Press [2nd] [F2], which is [F7]?. Select [1] Goal.		Press [ENTER] to close the screen.
Since one term is an integral, you need to select a sub-expression. To learn how to use the **Sub-expression Tool**, press [2nd] [F2], which is [F7]?. Select [3].		Read the instructions. Then press [ENTER].
Press [F3], which is Sub-expression Tool. It selects the first term. Press (▸) to select the second term which is an integral.		Write the integral here.

Techniques of Integration: *Symbolic Math Guide Practice & Summary*

Follow the instructions in this column.	The resulting screen	Use this colum to show your work. Use proper Calculus notation.
Press [F4] Transformation. Select [2] integration by parts.		Write the formula that you will apply.
Enter your choice for *f(u)* and *g'(u)*.		$f(u) =$ ________ $g'(u) =$ ________ $f'(u) =$ ________ $g(u) =$ ________
Press [ENTER] twice.		Verify that your substitutions agree.
Press [ENTER], You are in **Time to Think Mode.** This allows you to see the formula and think how it will be applied.		Apply the formula and write the resulting **equation**.
Press [ENTER] again. Press ▶ to see the entire expression. The formula has been applied and substituted into the sub-expression that was selected.		Substitute and write the new expression here.

At this point you may think that you are going in circles. You have arrived at an expression that contains the original integral.

1. Write an equation below that equates these two expressions in terms of the variable *u*.

$\int \left(e^u \cdot \cos(u)\right) du$ = ____

2. Substitute Y for the expression $\int \left(e^u \cdot \cos(u)\right) du$ in the equation above.

____

3. Solve for Y. ____

____

4. Back substitute the original expressions, in terms of *x*, into the equation in #3.

$y = \int cos(ln(x))dx$ = ______________________________

____

5. Exit **Symbolic Math Guide** by pressing [2nd] [ESC], which is [QUIT]. Calculate the integral $y = \int cos(ln(x))dx$ on the Home Screen. Did you get the same answer? Try to solve this example by a more direct route.

____

B. Use the Symbolic Math Guide to practice techniques of integration.

- To enter a new problem, press [F2] Problem, [1] New Problem.
- To exit the Symbolic Math Guide, press [2nd] [ESC], which is [QUIT].

Complete the examples that are given below.
- Write each step.
- Write the transformation that justifies each step.
- Observe patterns as you practice.

1. $\int (\ln(\sin(x)) \cdot \cos(x))dx$ ______________________________

2. $\int (x \cdot \sin(x))dx$ ______________________________

3. $\int (x \cdot \sin(x^2))dx$ ______________________________

4. $\int \left(\frac{\sin(\ln(x))}{x}\right)dx$ ______________________________

5. $\int (\ln(x)) \cdot dx$ ______________________________

6. $\int \frac{1}{(x-3)\cdot(x-2)}dx$ ______________________________

7. $\int \frac{2x-4}{(x^2-4x+3)}dx$ ______________________________

8. $\int \frac{(2x^2-5x+1)}{(x^2-4x+3)}dx$ ______________________________

9. $\int \frac{(x+2)}{(x^2+2x-3)}dx$ ______________________________

10. $\int \frac{x}{(x-a)\cdot(x-b)}dx$ ______________________________

Name:

Techniques of Integration: *Symbolic Math Guide Practice & Summary*

Journal Entries

1. This chapter discussed three techniques of integration. For each method do the following:

a. Name the method of integration and write the appropriate formula.

b. Explain why you would use that particular method of integration.

- How do you recognize the pattern?
- What hints or goals should you keep in mind?

c. Select an example that illustrates the technique.

- You may use an example out of your textbook that has **not** been assigned as homework.
  Write each step. Show all of your work.
  Write the transformation that justifies each step.

2. Write the following hint in your journal.
When selecting $u = f(x)$, look for functions in the following order: logarithmic functions, inverse trig functions, power functions, exponential functions, and then trigonometric functions. You can remember this order by the acrostic LIPET.

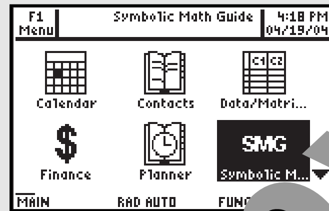
Name:

Chapter 7: *TECH-TIPS*

Lesson 32 TECH-TIPS

TECH-TIP | [APPS]: **Symbolic Math Guide: TI WEB Site**

This is a free **Flash Application** for your TI-89, which you can download from the TI WEB site.



If you do not have this **Flash Application** on your TI-89, you can download it from the TI WEB site http://education.ti.com. This is the Texas Instruments "educationportal" for Products, Training, Activities and Resources for You.

TECH-TIP | [APPS]: **Symbolic Math Guide: Installation**

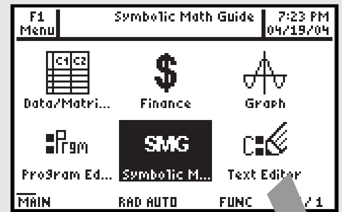
1. **New Batteries**
   - Put in a new set of batteries in your calculator.
     A word of caution! Before you begin any transfer to your calculator, install new batteries.
     Any interruption during the process is a major problem that you want to avoid. Once the download has begun, do not disconnect your calculator for any reason. If you do, the calculator will lock up. At that point you would need to consult your TI-89 Guidebook Appendix: "In Case of Difficulty". A word to the wise is sufficient. Don't go there. INSTALL NEW BATTERIES BEFORE YOU BEGIN.
2. **TI WEB site**
   - http://education.ti.com
   - This is the **Home** page of the TI Web site for Products, Training, Activities and Resources for You.
   - There is a pull-down menu on the right side entitled **Know what you're looking for?**
   - Click on the down arrow to see the choices.
   - You will need: Apps & OS versions
     TI Connect
3. **Download TI-89 Operating System - free**
   Download Symbolic Math Guide - free
   - On the **Home** page, click the down arrow from the menu **Know what you're looking for?**
   - On the pull down menu, select **Apps & OS versions**.
   - You will be directed to a new page.
   - Under "Select Your TI Technology" click on **TI-89/TI-89 Titanium.**
   - Scroll down and double-click on **Symbolic Math Guide**.
   - On the new page, read the directions carefully.
   - From this screen you can do the following:
     - Print detailed instructions for installing Apps.
     - Download the latest version of the **TI-89 Operating System**.
     - Download the **Symbolic Math Guide**.
     - Download a Symbolic Math Guide Guidebook.
     - View an animated illustration showing the solution of an equation using SMG.
   - Save the following files in a folder on your computer: **OS** and **SMG200**.
   - A folder named **TI Education** is created in the **Program Files** folder.
   - This is the default location: C:\Program Files\TI Education.
   - **Be sure to write down the location of the folder that you use so that you can locate it later.**
   - In the heading at the top, click on "Home".
4. **TI-Connectivity Kit - connecting your computer to your TI-89**
   - On the **Home** page, click the down arrow from the menu **Know what you're looking for?**
   - On the pull down menu, select **TI Connect.**
   - This replaces and improves upon the functions of TI-Graph Link.
   - Download the latest version of **TI Connect** software from this site.
   - Save the file to the default folder: C:\Program Files\TI Education.
   - Read carefully the instructions on the WEB site.
   - It is important that you install TI Connect **before** you connect your USB cable.
5. **Install TI Connect and Symbolic Math Guide**
   - To Install TI Connect in your computer, double-click on the file **ticonnect_eng**.
   - Use TI Connect to drag-and-drop the **smg** file onto the TI Device Explorer.

Chapter 7: *TECH-TIPS*

TECH-TIP [APPS]: Symbolic Math Guide: Using SMG: Detailed Instructions

Open SMG

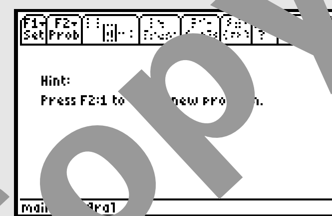
- [APPS] — Flash Applications
- Highlight **SMG** — **Symbolic Math Guide**
- Press [ENTER] — Your choices of **Problem Sets** are:
  1: **Current** to re-open the latest set.
  2: **Open**…to open an existing set.
  3: **New**…to create a new set.



Open Problem Set

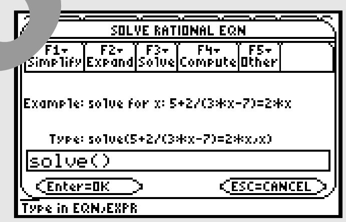
- ⊙ ⊙ [ENTER] — To create a New **Problem Set**
- Press ⊙ — Name the Variable: **integral**
- Press [ENTER] twice — To save the variable name.

Your screen displays the message: “Hint:
Press F2:1 to enter a new problem.

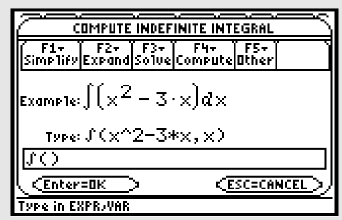


Enter a New Problem

- [F2], which is Problem, [1]New
  The calculator screen displays an example of the last type of problem that was entered into your calculator.
- The function key choices are:
  [F1] Simplify: [F2] Expand: [F3] Solve: [F4] Compute: and [F5] Other
  [F4] [1] for Derivatives, and [F4] [2] for Indefinite Integrals.

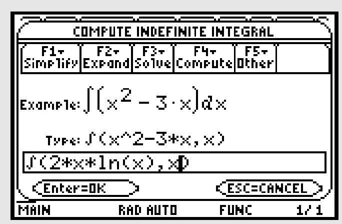


- Press [F4] Compute, [2] Indefinite Integral
  - **Example**: Illustrates correct mathematical form.
  - **Type**: Shows calculator syntax.
  - **Entry Line**: Provides an integral symbol and parentheses.

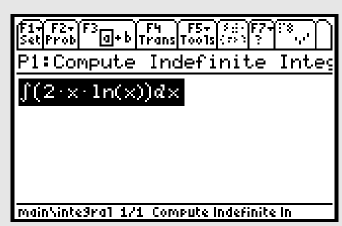


Enter and Execute the Command

- Enter the integrand 2 * x * ln(x), followed by a comma, and the independent variable x. The closing parenthesis is provided for you.



- Press [ENTER] after you have completed your entry for $\int 2 \cdot x \cdot ln(x)dx$.
- If you have made an error, press [F2] Problem, [2] Edit Problem.
  This takes you to the previous screen so that you can correct any errors.

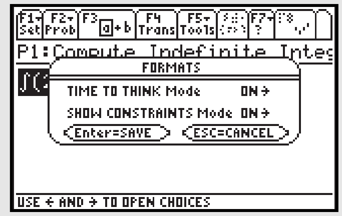


Chapter 7: *TECH-TIPS*

(Continued: SMG: Using Symbolic Math Guide)

Problem Set Format

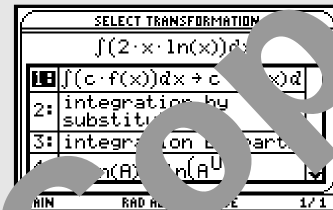
- [F1] Set, [9] Formats
  Time to Think Mode ON. Press ⊙ ⊙ [ENTER].
  Show Constraints Mode ON.
  This allows the calculator to introduce domain constraints when needed, such as when dividing by a variable which may take the value of zero.
- [ENTER] to return to the problem set.



Transformations

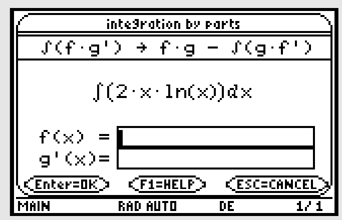
- [F4] Transformations
  A list of mathematical transformations appears. The list is appropriate for the type of problem set that you have selected.

  The **Symbolic Math Guide (SMG)** will allow you to do any operation that is mathematically "legal". You and another student may choose different paths to the same answer, or you may arrive at different forms of the correct answer. You should be able to recognize when two different forms are equivalent.

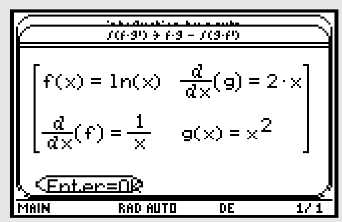


Integration by Parts

- [3] Integration by Parts
- The derivative of *u*, or *f*(*x*), should be simpler than *u*.
- dv or g'(x) must be easy to integrate. Look for part of the integrand that may be **the derivative** of some other function..
- Enter the expression for *f(x)* and *g´(x)* in the text box for each.
- Use the cursor pad, ⊙ or ⊙, to move between the text boxes.

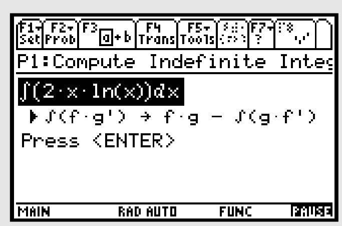


- Then press [ENTER] twice to see your choices displayed.
- Press [F1] Help on the previous screen as a last resort.

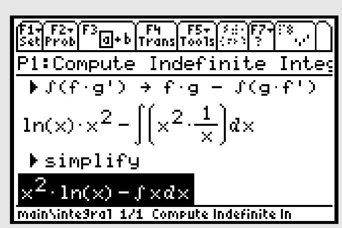


Time to Think Mode

- [ENTER] to see the formula below the original integral.
- In **Time to Think** Mode, only the formula is displayed.
- ***After each Transformation***, write the way that you think the rule will be applied.
- Write the transformation beside each step.



- [ENTER] to see the result displayed on the screen.
  Verify that your conjecture was correct.
- [ENTER] to automatically simplify the algebraic expression in the integrand.



Chapter 7: *TECH-TIPS*

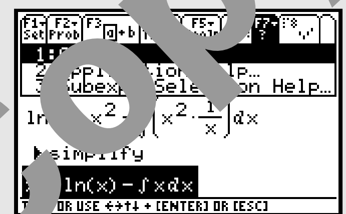
TECH-TIP [APPS]: Symbolic Math Guide: Simplification Rules

When applying the formula for integration by parts in **Symbolic Math Guide**, the order is dictated by the choice of *f(x)* and *g(x)*. However, in the next step *simplify*, the factors of the first term are reversed.

The **Computer Algebra System (CAS)** built into the TI-89 presents solutions by certain simplification rules. Variable factors are in alphabetical order followed by functions such as *ln(x)*.

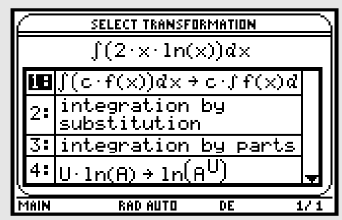
TECH-TIP [APPS]: Symbolic Math Guide: [F7] Goal

- [2nd][F2], which is [F7]?. Your choices are shown on the screen.
- [1] Goal. The stated goal is to remove all indicated integrals and then simplify.
- [ENTER] to close the screen.

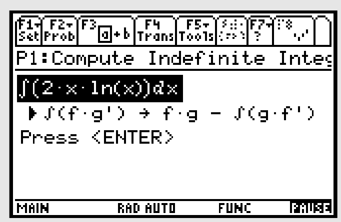


TECH-TIP [APPS]: SMG: [F1] Set: [9] Format: Time to Think Mode

- [F4] Transformation. Scroll down to see the list of possible choices.
- Highlight the rule that you prefer and press [ENTER], or press the number of your choice.



- In **Time to Think** Mode, only the formula is displayed.
- Predict the outcome of applying the rule that you have selected.
- Write the predicted result for each step on your paper.
- Beside each step, write the Transformation that you used.
- Press [ENTER] to see the result.



If you're going to play Math, you've got to learn the rules!

Name:

Chapter 7: *TECH-TIPS*

TECH-TIP [APPS]: **Symbolic Math Guide:** [F2] **Problem:** [1] **New Problem**

[F2] Problem, [1] New Problem

- You are back at the **Compute Indefinite Integral** screen.
- Enter another integral and proceed as outlined in the example above.

- Other options available are:
  [F1] Simplify, [F2] Expand, [F3] Solve, [F4] Compute, and [F5] Other
- If you press any of the function keys you will see a sub-menu.
- Press [ESC] to close the menu.

TECH-TIP [APPS]: **Symbolic Math Guide: [QUIT] To Exit SMG**

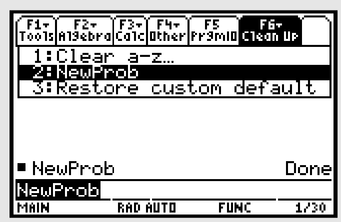
To exit the Symbolic Math Guide, press [2nd] [ESC], which is [QUIT].

Lesson 33 TECH-TIPS

TECH-TIP [HOME] **Screen: [F6] CleanUp:** [2] **New Problem**

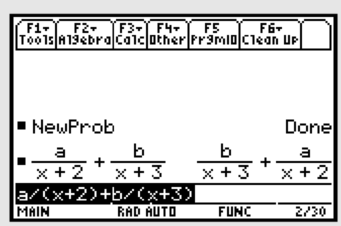
As always, begin each section with a **NewProblem** command.

- [2nd] [F1], which is [F6], [2] [ENTER].



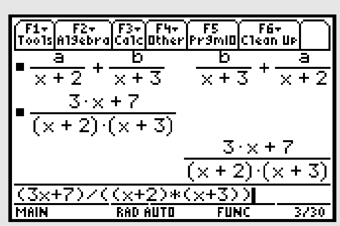
TECH-TIP [HOME] **Screen: Entering Fractions**

- Enter the fractions as shown on the screen.
- Then [ENTER].
- Remember that the fraction bar is a grouping symbol.
  Therefore, the expressions in the numerator or in the denominator need to be enclosed by a pair of parentheses.



Example:

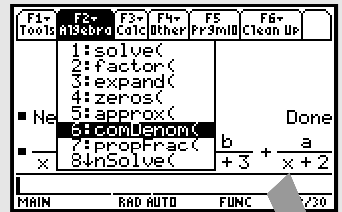
- The TI-89 has no implied multiplication.
- It is always a good idea to use a multiplication operator when multiplication is intended.

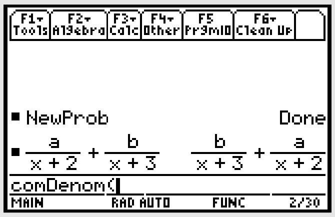


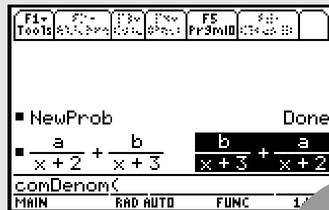
Chapter 7: *TECH-TIPS*

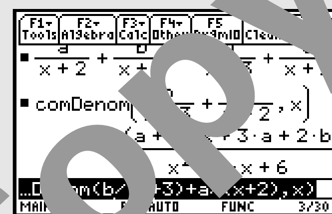
TECH-TIP [HOME] Screen: [F2] Algebra Menu: [6] Common Denominator

- [F2] Algebra, [6]Common Denominator
- ⏶ Highlight the previous expression
- [ENTER] to paste the expression in the Entry Line.
- [,] comma: [X] with respect to the variable *x*: [)] close parenthesis.
- [ENTER] to execute the command.



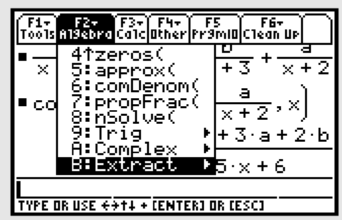


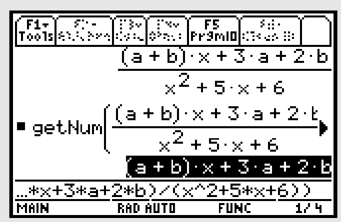




TECH-TIP [HOME] Screen: [F2] Algebra Menu: B: Extract Numerator

- [F2] Algebra ⏶ B: Extract
- ⏵ to see Menu [1] get Numerator(
- ⏶ to highlight the previous expression on the History area
- [ENTER] to paste the expression into the Entry Line
- [)] close parenthesis
- [ENTER] to execute the command.





TECH-TIP [HOME] Screen: [F2] Algebra Menu: [1] Solve a System of Equations

- Press [F2] Algebra, [1] Solve.
- Then type "$a + b = 3$". Press [alpha] [=] for "*a*" and [alpha] [(] for "*b*".
- Press [CATALOG] [=], which takes you to the first command in the Catalog beginning with the letter "a".
- Press ⏷ to move the indicator to the word "and". Then press [ENTER].
- Type the equation "$7 = 3 \cdot a + 2 \cdot b$".
- Complete the command with [,] [alpha] [=], which is *a*, [)] [ENTER].