

Applications & Differential Equations: *Hubble's Law & Homo Sapiens*

## Lesson 35

**Tycho Brahe** spent his life trying to answer the wrong question. Did the earth circle the sun or was the earth the center of the universe? Even though he suspected that the earth went around the sun, he never could prove it. His conclusion was correct, but his basic premise, that the path was a circle, was incorrect. Therefore, he never could get his data to fit his mathematical model.

His assistant, **Johann Kepler**, held him in such awe that it was years after Brahe's death before Kepler discarded the circular models. Kepler later arrived at the correct conclusion that the planets travel in elliptical orbits around the sun.

Tycho Brahe spent over 20 years of intense record keeping and observation of the heavenly bodies. Johann Kepler devoted his life to experimentation and analysis of that data.

At one point, Kepler had derived a model of the orbit for Mars which satisfied Brahe's data to within 8' of an arc. He was so convinced of the accuracy of the data that he decided to start over from the beginning. Kepler's investigations led to his three Laws of Planetary Motion.<sup>1</sup> He did all of this without the powerful tools of calculus, a computer or a calculator.

This chapter explores the use of integral calculus in the real world. Many vocations use the power of calculus to solve problems. This lesson will investigate a few of them.

### A. Hubble's Law

It is fitting that the Hubble space telescope, which sends pictures from outer space back to earth today, was named for **Edwin Hubble**. In the 1920's, his observations led to the fascinating discovery that all galaxies are moving away from us. This is known as Hubble's Law and states that nearly all galaxies are moving at a velocity that is proportional to their distance,  $R$ , from the Earth. The constant of proportionality is  $H$ , the Hubble constant setting the rate of expansion.<sup>2</sup>

Given Hubble's Law:

$$V = H \times R \text{ where}$$

$V$  = Velocity of the galaxy traveling away from Earth,

$H$  = Hubble's constant, and

$R$  = the distance from the Earth to the galaxy.

Find a model,  $R(t)$ , for the distance from the Earth to the galaxy. Ignore units in this problem.

1. Solve analytically. Show all of your work.

<sup>1</sup> Flower, Phillip, Clemson University, 1990. West Publishing Company, 50 W. Kellogg Boulevard, P.O. Box 64526, St. Paul, MN 55164-1003, p. 93-102.

<sup>2</sup> Hawking, Stephen, 2001. *The Universe in a Nutshell*, 2001. Bantam Books, 1540 Broadway, New York, New York 10036, p. 77.

2. Check using  $\boxed{F3}$  C:deSolve on the  $\boxed{HOME}$  Screen.

**TECH-TIP** $\boxed{HOME}$  Screen:  $\boxed{F6}$  Clean Up:  $\boxed{2}$  NewProblem

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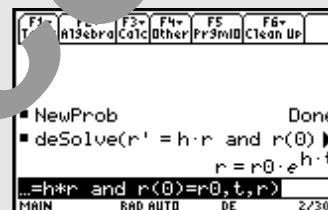
**TECH-TIP** $\boxed{HOME}$  Screen:  $\boxed{F3}$  Calculus Menu: C:deSolve(

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**deSolve** ( $r' = h \cdot r$  and  $r(0) = r_0, t, r$ )

- This translates: Solve the differential equation for  $r$  when  $r$ -prime is the product of the constant  $h$  and the variable  $r$ . The initial conditions are that  $r = 0$  when  $t = 0$ . The independent variable is  $t$ . The dependent variable is  $r$ .

**Note:** The initial condition is “ $r$  of zero equals  $r$  zero”. There is no implied multiplication. This is not meant to be the letter *alpha-o*. It is a calculator two-letter variable name,  $r_0$ , representing the initial value,  $R_0$ .

**B. Homo Sapiens**

For 99.9 percent of the time that Homo Sapiens have existed, the total population was less than 10 million people. In the last 200 years the growth has become exponential and the world's population is expected to double every forty years.<sup>3</sup> In 1880 the population reached its first billion. It only took 130 years to reach the 2nd billion in the year 1930. The current population is 6.3 billion.<sup>4</sup>

**Dr. Stephen Hawking**, Professor of Mathematics at the University of Cambridge and theoretical physicist, has written two bestsellers. In *The Universe in a Nutshell* he predicts that by the year 2600 the world's population will be standing shoulder to shoulder.<sup>5</sup>

The National Wildlife Federation urges us to consider the fact that, “we live on a planet with finite resources—resources that need to be shared by an unprecedented number of humans. Six billion humans sharing the earth is unlike anything ever experienced in human history and is having devastating impacts on our environment and natural resources.”<sup>6</sup>

What model would you choose to fit the population data? Explain why you chose it.

<sup>3</sup> Hawking, *The Universe in a Nutshell*, p. 158.

<sup>4</sup> *Global Population Growth: The Numbers and What They Mean*, National Wildlife Federation, 1400 16<sup>th</sup> Street, NW, Suite 501, Washgton, DC 20036. <http://www.nwf.org/population/>

<sup>5</sup> Hawking, p. 159.

<sup>6</sup> *Global Population Growth: The Numbers and What They Mean*, p. 2.

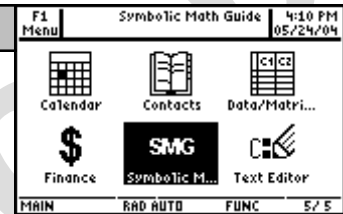
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2. Write a differential equation that states the following:  
The rate is proportional to the population,  $y$ , at a given time,  $t$ , and to the difference between the population and its limiting population,  $P$ . Let  $K$  be the constant of proportionality.
3. Solve the differential equation.
- Use the parameters given in the differential equation.
  - You may use the TI-89 **Symbolic Math Guide** to assist you.
  - However, you should use the **Time to Think** feature to write your solution before you let the calculator show you the answer.
  - Write each step neatly and clearly.

**TECH-TIP****[APPS]**: Symbolic Math Guide: Using SMG

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4. It took from the beginning of the human species until around 1800 to reach a population of 1 billion, until 1930 to reach 2 billion, until 1960 to reach 3 billion, until 1974 to reach 4 billion, until 1987 to reach 5 billion, and until 1999, 6 billion.<sup>7</sup> According to the U.S. Census Bureau, the world's population is 6.377 billion in 2004.<sup>8</sup>

- Use the TECH-TIP below to create a **Data File** named **worldpop**.
- Then use a **Regression Equation** to model the data.
- Refer to the pages within each TECH-TIP for further instructions.

F1= Tools	F2 Plot Setup	F3 Cell	F4 Header	F5 Calc	F6 Util	F7 Stat
DATA						
	c1	c2	c3			
1	1880	1				
2	1930	2				
3	1960	3				
4	1974	4				
r1c2=1						
MAIN RAD AUTO FUNC						

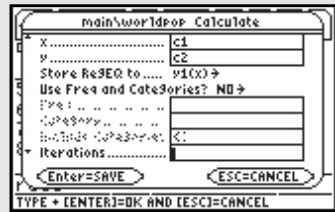
**TECH-TIP** [APPS]: Data/Matrix Editor: Create a New Data File page 252

- Create a **Data File** named “**worldpop**”.
- Let column 1 represent the years. Use all 4 digits.
- Let column 2 represent the population in billions.



**TECH-TIP** [APPS]: Data/Matrix Editor: [F5] Calculate a Regression Model page 253

- Calculate a Regression Equation using [F5] Calculate Menu.
- Use a **Logistic** model for the data in (c1, c2).
- Store the Regression Equation in  $y_1(x)$ .



5. Record the resulting equation in the space below.

$y_1(x) =$  \_\_\_\_\_

**TECH-TIP** [APPS]: Data/Matrix Editor: [F7] Statistical Variables page 254

After you have done a regression, the statistical parameters are stored under [2nd][F2], which is [F7] **Stat Vars**. If you leave the **Data/Matrix Editor**, this information is no longer stored in this location.

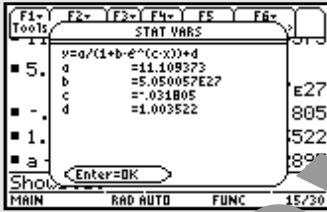
<sup>7</sup> *Global Population Growth: The Numbers and What They Mean*, p. 1.

<sup>8</sup> <http://www.census.gov/ipc/www/worldpop.html>

However, they are stored on the [HOME] Screen in the Math Menu under Statistics. See the TECH-TIP below.

**TECH-TIP** [HOME] Screen: [MATH] Menu: [6] Statistics: [8] ShowStat Variables page 254

- Press [HOME] [CLEAR].
- Press [2nd] [5], for the [MATH] Menu.
- Select [6] Statistics,
- Then press [8] **Show Stats** to paste the command on the **Entry Line**.
- Press [ENTER] to see the **Stat Vars** Screen.
- To exit this screen, press [ENTER].



6. Record the values for each parameter.

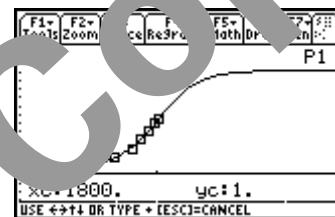
- Store the values for each parameter. Do not round off.
- For example: Enter the value of the parameter, then press [STO>] [alpha] [=] for *a*.

*a* = \_\_\_\_\_

*b* = \_\_\_\_\_

*c* = \_\_\_\_\_

*d* = \_\_\_\_\_



7. Sketch the graph of the equation,  $y_1(x)$ , and Plot 1 in the space provided.

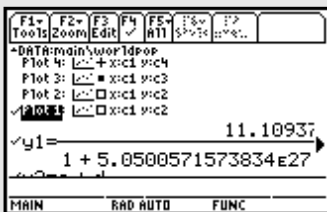
- Use the TECH-TIP below to define Plot 1 in the [Y=] Editor.
- Show a complete graph of  $y_1(x)$  and all points of Plot 1.
- Indicate the Window that you used.

**Complete Graph:** A complete graph of a function shows important features of the graph such as: extrema, concavity, and end behavior.

It may be necessary to have two viewing windows to view all features of a complete graph.

**TECH-TIP** [Y=] Editor: Define a Data Plot from the [Y=] Editor page 255

- Press [F1] to access the [Y=] Editor.
- Press [C] to move the cursor to  $y_1=$ .
- Highlight Plot 1, press [ENTER] to define **Data Plot 1**.
- Define Plot 1 as a **Scatter Plot** using a **Box Mark** for the data points ( $\square$  or  $\square$ ).
- Press [2nd] [F1] to return to the [Y=] Editor.



**TECH-TIP** [Y=], [WINDOW] or [GRAPH]: [F2] Zoom: [9] Adjusting ZoomData page 256

Window: \_\_\_\_\_



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8. According to this model, what will the population be in the year 2100?

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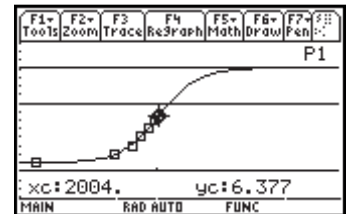
9. How many people will there be in the year 2600 when Stephen Hawkins predicts that we will be standing shoulder to shoulder?

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10. According to this model, what is the carrying capacity of the earth?
- Show your work in the space below.
  - Use the parameters that you have stored in your calculator to evaluate the limit.
  - Enter the value of the carrying capacity as  $y2(x)$  in the [Y=] Editor.

11. There is some evidence that the modernization of today's society has caused a decrease in the overall birth rate. Some experts believe that the population growth rate is declining and that within the next fifty years the population will peak at about 8 billion.<sup>9</sup>

- Let  $y3(x) = 8$  in the [Y=] Editor.
- According to your model, during what year will the population reach 8 billion?
- Show your work. You may use the [F5] Math Menu.
- Write your answer in a complete English sentence.




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12. Find the point of inflection of this model.

- You may use [F5] Math, [8] Inflection from the Graph Application.
- Justify your answer using correct mathematical grammar.
- Mark the point of inflection on the graph in #11.

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<sup>9</sup> Singer, Max, The Population Surprise, *The Atlantic Monthly*, August 1999. <http://www.theatlantic.com/issues/99aug/9908popdrop.htm>

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13. Explain the significance of the point of inflection in the context of the population of the Earth.

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14. What are some things that could cause a decrease in the rate of population growth?

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15. According to the model will the Earth's population ever reach 12 billion?  
State your answer in a complete English sentence.

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**TECH-TIP**

[HOME] Screen: [F2] Algebra Menu: [1] Solve( Solve an Equation

Solve( $y1(t)=12,t$ )

- Use [HOME] Screen, [F2] Algebra Menu, [1] Solve(
- Enter the equation, followed by a comma and the variable you are solving for, then [)].
- Press [ENTER] to see the solution.

Dr. Hawking concludes, "There is a sick joke that the reason we have not been contacted by extraterrestrials is that when a civilization reaches our stage of development, it becomes unstable and destroys itself. However, I'm an optimist. I don't believe the human race has come so far just to shut itself out when things are getting interesting."<sup>10</sup>

<sup>10</sup> Hawking, p. 159.