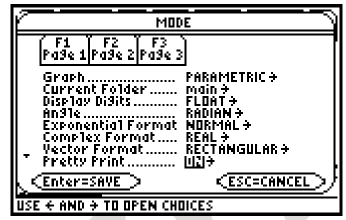
Name:

Parametric, Polar & Vector Equations: *Parametric Transformations & Derivatives*

A function defines a dependent variable *y* in terms of an independent variable *x* so that for every *x*-value there is one, and only one, value of *y*. We say, then, that *y* is a function of *x* and write $y = f(x)$.

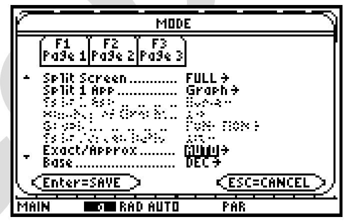
In parametric form, both of the variables *x* and *y* are defined in terms of the **parameter** *t*. Any function $y = f(x)$ may be written in parametric form as follows:

$x(t) = t$
$y(t) = f(t)$

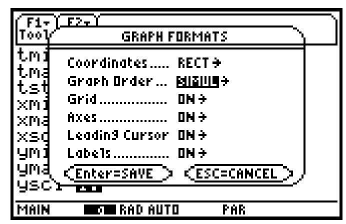


Calculator Setup

- [HOME] Screen: **New Problem**
- [MODE]: Change Graph Mode to **Parametric**.
- Angle Mode: **Radian**.



- [F2] On page 2: **Auto Mode**
- Press [ENTER] to save the changes and exit the Mode screen.



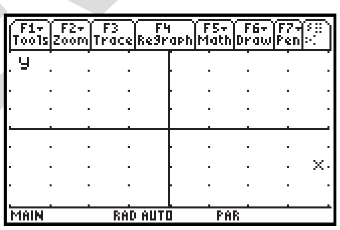
- [◆] [F3], [GRAPH] Screen: [F1] Tools, [9] Graph Formats:
- Adjust so that your screen matches the one shown.
- [ENTER] to save and exit.

A. Transformations and Derivatives

1. In this exploration, $f(x) = \sin(x)$.
Define this as parametric equations $x(t) = t$, and $y(t) = f(t)$.

- In the [Y=] Editor, [F1] [8] Clear Functions.
- Define $xt1(t) = t$ and $yt1(t) = \sin(t)$.
- One complete period of the sine function would be from $t = 0$ to $t = 2\pi$.
- [WINDOW]: T [0, 2π, 0.1] X [-2π, 2π, π/2] Y[-4, 4, 1].

a. Sketch the graph in the space provided below.
- [◆][F3], for [GRAPH].

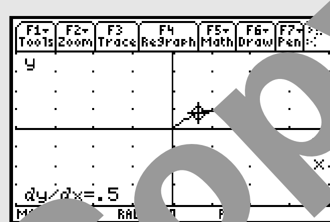


In this section you will explore the effect of adding and multiplying parametric equations by constant values.

- How does it affect the graph?
- How does it affect the equation in function notation?
- How does it affect the derivative?

b. Find the derivatives of $xt1(t) = t$ and $yt1(t) = \sin(t)$ at $\frac{\pi}{3}$ using the [F5] **Math Menu** on the [GRAPH] Screen.

TECH-TIP [GRAPH] **Screen: [F5] Math Menu: [6] Derivatives:Parametric:** ***dy/dx−dy/dt−dx/dt***



- [F5] Math Menu, [6] Derivatives
- Select [1] for $\frac{dy}{dx}$;
- The calculator needs to know the value of t to use in evaluating the derivative that you have selected. The question (dy/dx at?) is asking you to input the t-value, $\pi/3$.
- Then press [ENTER] to begin the calculation.
- The answer will appear at the bottom of the screen.
- Repeat the process for [2] $\frac{dy}{dt}$; and [3] $\frac{dx}{dt}$.

Note: When you have more than one graph on the screen, press ⊝ to move to the correct graph before you input the specific t-value. Then press [ENTER] to execute the command.

Record your answers here: 1. $\frac{dy}{dx}$ = ________ 2. $\frac{dy}{dt}$ = ________ 3. $\frac{dx}{dt}$ = ________

For # 2 – 7, follow these directions:

a. Sketch the graph of $(xt1(t), yt1(t))$ and $(xt2(t), yt2(t))$ in the space provided.
- Let $xt1(t) = t$ and $yt1(t) = \sin(t)$.
- Make the Graph Style of $(xt1, yt1)$ **Thick** to distinguish the parent function from the transformed functions.
- Let $0 \le t \le 2\pi$.
- The definition for $(xt2, yt2)$ will be given for each example.

TECH-TIP [Y=] **Editor: [F6] Graph Style** *page 298*

TECH-TIP [Y=] **Editor: [F3] Edit** *page 298*

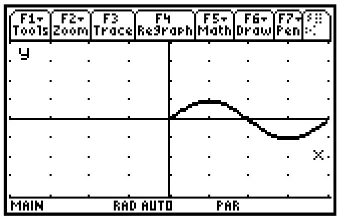
b. Write the parametric equations in function notation, $y = g(x)$.
- For each pair of parametric equations, solve the $x(t)$ equation for t and substitute into the $y(t)$ equation to obtain a function $y = g(x)$.

c. Find $\frac{dy}{dx}$, $\frac{dy}{dt}$, and $\frac{dx}{dt}$ at $t = \frac{\pi}{3}$ using the Math Menu from the Graph Screen.

d. Complete the statement at the end of each example.

Addition = Shifts

2. Vertical Shift
$xt2(t) = t$
$yt2(t) = \sin(t) + 1$

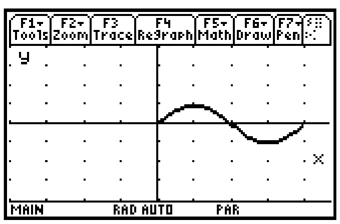
a. 

b. $g(x) =$ _________

c. $\frac{dy}{dx} =$ _________ $\frac{dy}{dt} =$ _________ $\frac{dx}{dt} =$ _________

d. Shifting the graph _________________ one unit (did or did not) ___________ affect the derivative.

3. Horizontal Shift
$xt2(t) = t + 1$
$yt2(t) = \sin(t)$

a. 

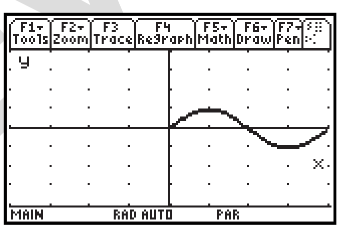
b. $g(x) =$ _________

c. $\frac{dy}{dx} =$ _________ $\frac{dy}{dt} =$ _________ $\frac{dx}{dt} =$ _________

d. The derivatives were identical because they were calculated for the same values of the variable _______ and, therefore, on the same relative ______________________________ on the graph.

Multiplication = Dilation

4. Vertical Dilation
$xt2(t) = t$
$yt2(t) = 2 \cdot \sin(t)$

a. 

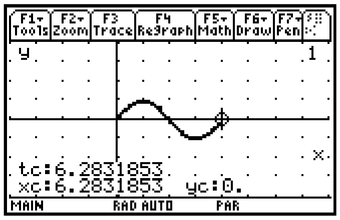
b. $g(x) =$ _________

c. $\frac{dy}{dx} =$ _________ $\frac{dy}{dt} =$ _________ $\frac{dx}{dt} =$ _________

d. A vertical stretch by a factor of 2 makes the derivatives ______________________________ twice as large.

5. Horizontal Dilation (Hint: Double Xmax in the viewing window.)
$xt2(t) = 2 \cdot t$
$yt2(t) = \sin(t)$

a.



b. $g(x) =$ _________

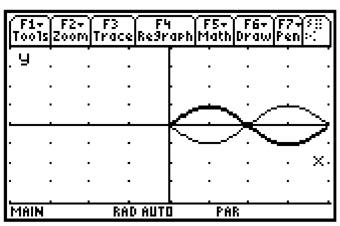
c. $\frac{dy}{dx} =$ _________ $\frac{dy}{dt} =$ _________ $\frac{dx}{dt} =$ _________

d. The horizontal factor of 2 makes Δx __________________ for each t-step; therefore the period of the sine function is _____________________________ its normal period.

Opposite = Reflection

6. Vertical Reflection
$xt2(t) = t$
$yt2(t) = -\sin(t)$

a.



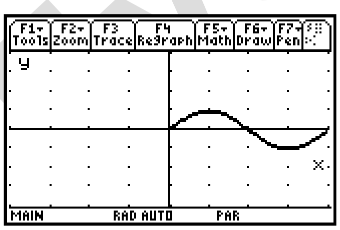
b. $g(x) =$ _________

c. $\frac{dy}{dx} =$ _________ $\frac{dy}{dt} =$ _________ $\frac{dx}{dt} =$ _________

d. A vertical reflection makes the derivatives _________ and __________ the __________________ of the derivatives of the first function.

7. Horizontal Reflection
$xt2(t) = -t$
$yt2(t) = \sin(t)$

a.



b. $g(x) =$ _________

c. $\frac{dy}{dx} =$ _________ $\frac{dy}{dt} =$ _________ $\frac{dx}{dt} =$ _________

d. Write an equation that states the relationship between $\frac{dy}{dx}$, $\frac{dy}{dt}$, and $\frac{dx}{dt}$ that is true for every example above.

____

Journal Entries

1. When adding a constant ***c***, does every point move the same amount? Justify your answer.

2. When multiplying by a constant, what points on the graph remain the same?

3. A vertical reflection reflects the graph across the ______ -axis. Justify your answer.

4. A horizontal reflection reflects the graph across the ______ -axis. Justify your answer.

5. Explain why all horizontal transformations appear as inverses in the resulting equation in function form.

6.
- **a.** Write the equation you stated as your conclusion in #7d, Part A.
- **b.** Solve part (a) for $\frac{dy}{dt}$.
- **c.** The equation should remind you of a calculus rule that you have used many times. What is the name of that rule?
- **d.** State that rule using function notation instead of differentials. Use the functions and variables of your equation in #6b above.
- **e.** Write the rule in English words by completing the following statements:
  The derivative of a composite function is the product of ____.
  The derivative of the outside function must be evaluated ____.

7. Given: $y = \sin(3x)$
In light of the rule that you have just stated, answer the following questions:
- **a.** Explain what the transformation is and how it affects the graph of $y = \sin(x)$.
- **b.** How does the transformation affect the derivative $\frac{dy}{dx}$?
- **c.** Interpret this transformation in the context of a particle traveling along a rectilinear path.