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## Introduction

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**Exploring Geometry Through Puzzles and Games MathSet** includes 34 enrichment activities that help students develop spatial ability, logical reasoning, and an understanding of geometric properties and relationships.

These activities

- have an easy entry point and may be explored on many levels by students of different ages and aptitudes;
- offer options for further investigation;
- require a minimum of preparation;
- may be incorporated into a regular math class, a math club, or an enrichment program;
- are appropriate for large and small learning groups, and partner or independent work; and
- enable students to enjoy mathematics with their families when assigned as homework.

### The NCTM Standards

In Standards 2000, the National Council of Teachers of Mathematics calls for an active, hands-on approach to the study of geometry throughout the elementary and middle school grades.

Specific focus is placed on

- analyzing characteristics and properties of geometric objects;
- understanding the usefulness of transformations and symmetry in analyzing mathematical situations;
- using visualization and spatial reasoning to solve problems; and
- relating the study of geometry to other topics in mathematics, including number, measurement, and pattern.

This MathSet was developed to help teachers implement these standards.

## USING THIS MATHSET IN THE CLASSROOM

### Organization and Sequence

The activities are organized into six sets according to materials: Tangram, Sphinx Puzzle, Pentominoes, Hexiamonds, Designs and Edge Matching, and Dots and Shapes. Within each activity set, students explore a variety of concepts, including size, shape, properties of polygons, congruence, similarity, symmetry, and area. In addition to some knowledge of these concepts, students should have some familiarity with the following geometric terms: polygon, parallelogram, trapezoid, square, rectangle, hexagon, and triangle (equilateral, isosceles, scalene). There is no recommended sequence for presenting the six sets of activities. However, certain activities do build on previous ones within each activity set.

Teachers can select materials and activities which they believe will best meet their instructional goals. For example, a teacher may choose to work with one material to explore a variety of different concepts, or may focus on one concept, such as symmetry, and use a variety of different materials to enrich students' understanding.

### Class Discussion

Students develop thinking skills by articulating their ideas and by observing how other people think. Teachers are encouraged to provide opportunities for students to listen to each other's problem-solving approaches. The activities encourage creative thinking by presenting problems and puzzles that have multiple solutions, or are open-ended explorations.

### **Teacher Notes and Solutions**

An example of one possible solution is generally illustrated on the Activity page. Where specific results are expected, they are detailed for the teacher's reference in Teacher Notes and Solutions (pp. 3-17).

### **Activities and Materials Masters**

The pages for duplication are divided into two categories: Activities and Materials Masters.

Activity pages may be used in several ways:

- Laminate copies of the Activities for repeated use and ask students to record their answers and work on a separate sheet of paper.
- Duplicate copies for students to use as worksheets.
- Create transparencies for presentation to a large student group.

The Materials Masters are intended to be copied for use with the Activities. Duplicate the Masters on construction paper or card stock for greater durability and ease in manipulation. The "You Need" section found at the beginning of each Activity page specifies which Materials Masters are required for that activity.

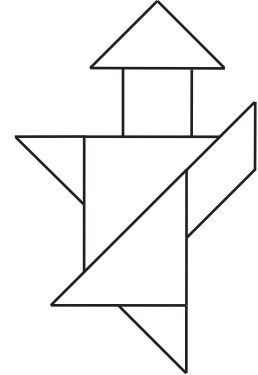
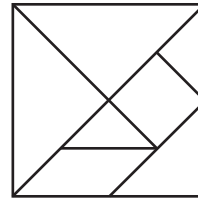
Students should be instructed to save their puzzle pieces (tangram, Sphinx Puzzle, pentominoes, etc.) for use in several activities. Teachers may wish to duplicate, cut out, and store some extra sets of each kind of puzzle to have on hand for classroom use.

Other standard materials which may be used in activities include scissors, unlined paper, a ruler, and colored pencils, crayons, or markers. The Pentominoes 6 activity titled Reflection Puzzles (page 17) requires a small rectangular mirror.

## Creating Tangram Puzzles

**You Need:** Tangram Master duplicated on colored construction paper or card stock, scissors, and drawing paper

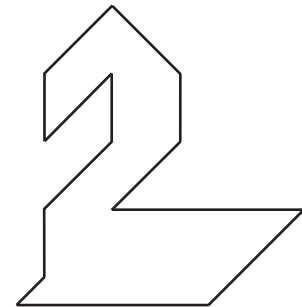
A **tangram** is a set of seven shapes cut from a square. The traditional way to play tangram is to arrange the pieces to form objects: animals, people, everyday objects, letters, numbers, geometrical shapes, and so on.



### Tangram Rules:

- Use all seven pieces for each shape.
- Each piece must touch another piece at one or more points.
- The pieces must not overlap.

1. Carefully cut out a set of tangram pieces from one square on a copy of the Tangram Master.
2. Explore making different shapes using all seven tangram pieces.
3. When you find a shape you like, draw the outline of the shape on drawing paper.
4. Write a title for your shape, and sign your name.
5. Challenge other students to solve your puzzle by fitting the tangram pieces into your outline. Successful puzzlers may sign their names on the back of the puzzle.
6. Save your tangram pieces for the next activities.



*Swan by Brian*

### Background

Tangrams come to us from China, but we don't know when they were first created. The earliest Chinese book about tangrams was produced around 1800, but it seems certain that tangrams were already old by then.

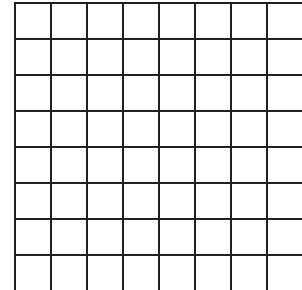
Tangram puzzles became very popular in Europe and America in the nineteenth century. Lewis Carroll, the British author of *Alice in Wonderland*, loved to play with tangrams. Many other famous people enjoyed tangrams, including French emperor Napoleon Bonaparte, American writer Edgar Allan Poe, and U.S. President John Quincy Adams.

## Pentomino Games

**You Need:** 1-inch Squared Paper Master, one or two sets of pentominoes, construction paper or card stock, a ruler

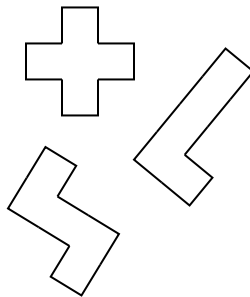
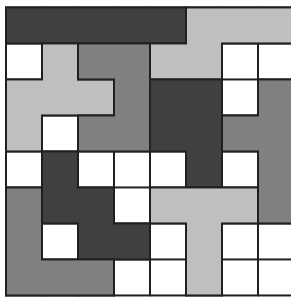
### Preparation

Draw a game board on construction paper or card stock. The board is an 8-by-8 array of 1-inch squares. (You may cut out the array from two sheets of 1-inch squared paper and mount it on card stock.)



### Basic Game

There are two players. Place all the pentominoes between you. Decide who goes first. The other player starts the next game. Take turns choosing a pentomino and placing it on the board. Pentominoes must cover whole squares and they may not overlap. Once a pentomino is on the board, it may not be moved. The first player who is unable to place a pentomino on the board loses the game.



*It is not possible to place any of the remaining pentominoes on this board.*

### Variations

- Take turns choosing pentominoes until each player has six. Then play the basic game. If one player cannot lay down a pentomino, the other player can keep playing. The player with fewer pentominoes left is the winner. A game may end in a tie.
- Play with two complete sets of pentominoes. Each player starts with one complete set.
- Play on a game board of a different size or shape. For example, draw a 6 x 10 array of 1-inch squares, forming a rectangular grid.
- Play as a cooperative game. The object is to place as many pieces as possible on the game board before getting stuck. Try playing without talking.
- Combine the variations, or make up your own.

## Nine-Dot Polygons Bingo

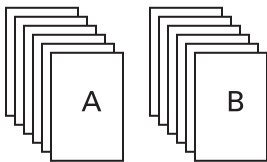
**You Need:** a set of nine-dot polygons from the Dots and Shapes 1 activity titled Nine-Dot Polygons, scissors, 5 identical game pieces for each player that are distinguishable from the pieces of the other player

*Set of Game Cards*

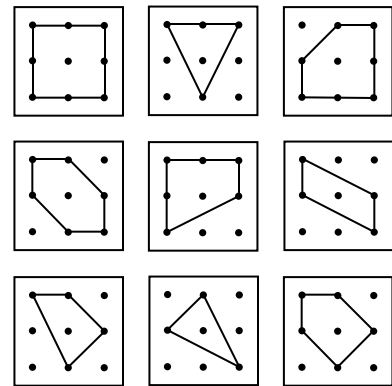
<b>A</b> not a quadrilateral	<b>A</b> quadrilateral	<b>A</b> polygon	<b>A</b> more than 4 sides	<b>A</b> area $\leq 2$ square units	<b>A</b> area $> 2$ square units
<b>B</b> no parallel sides	<b>B</b> no obtuse angles	<b>B</b> no acute angles	<b>B</b> at least 2 right angles	<b>B</b> no line of symmetry	<b>B</b> exactly 1 line of symmetry

**Preparation:**

Cut out the cards above and place them face down in two piles, A and B.



If you have not already done so, cut out the 14 nine-dot polygon shapes from the Dots and Shapes 1 activity titled Nine-Dot Polygons. Arrange any nine of them in a 3 x 3 array.



*Sample Playing Board*

**Directions:**

Players take turns. The first player chooses one card from each pile. He or she covers with a game piece any polygon that matches the information on both cards. If a player cannot find a polygon that matches both cards, he or she loses a turn.

When all the cards have been used, mix them up and place them back in the two piles.

The first player to place three of his game pieces on the board in a horizontal, vertical, or diagonal row wins.