Listed here are the Explorations. Following is a "Road Map" which has been included to give you a sense of the concepts and how they interrelate. The approximate time, given in days, assumes a 40–50 minute class period and some work done outside of class, perhaps 20–30 minutes. The approximate time is the number of days expected to complete the problems and activities and to discuss the results and ideas. It does not include any time for assessments such as quizzes or tests. The times should be thought of as a relative guide, as for any set of students, school, or even time of year, they will vary.

Two of the Explorations are divided into two sections. The approximate times listed for these refer to both components.

Exploration	**Contents**	**approximate time (days)**
Exploration 1:	Polygons and their Angles	1.0 – 1.5
Exploration 2:	The Vertex Packing Problem	2.0
Exploration 3:	Analyzing Complete Vertex Packings	2.0
Exploration 4:	Tessellations and Duals	1.5 – 2.0
Exploration 5:	Incomplete Vertex Packings: Folding into 3-Space	2.0 – 3.0
Exploration 5 Cont.:	More on Nets	
Exploration 6:	Fuller Design Problems	1.5 – 2.0
Exploration 7:	The Five Platonic Solids	1.5
Exploration 8:	Schlegel Diagrams	1.5
Exploration 9:	Euler's Formula	1.5
Exploration 10:	Duals in Space	1.0
Exploration 11:	Some Semi-Regular Polyhedra: Prisms & Antiprisms	2.0 – 3.0
Exploration 11 Cont.:	Prisms & Antiprisms: Working with their Numbers	
Exploration 12:	Other Semi-Regular Polyhedra	2.0
Exploration 13:	Truncations of Platonic Solids	1.5
Exploration 14:	Duality and the Archimedean Solids	2.0 – 3.0
Exploration 15:	Double-Truncations and the Snubs	2.0 – 3.0
		total: 25.0 – 30.5

These Explorations have been designed so that there is a constant building process taking place. The richness and interconnectedness of the material often means that ideas are sometimes introduced, but not fully addressed at that time. It is OK for students to not fully understand everything that is happening in an Exploration before moving on to the next, as they will likely see the material again. Often in a subsequent Exploration, a more detailed analysis of the mathematics will be found. Some sort of closure is necessary, however, and each Exploration takes up a general problem or idea with which students should feel comfortable by the end of discussion. The lingering questions serve to motivate a search for a greater understanding.

The Explorations have generally been written for individual or small group work. There is no need to use this format exclusively. Depending on your students and the material, you may want to present a particular topic to the large group or pose a couple of problems to the class. Improvise to fit your needs.

Enjoy!

EXPLORATION 5

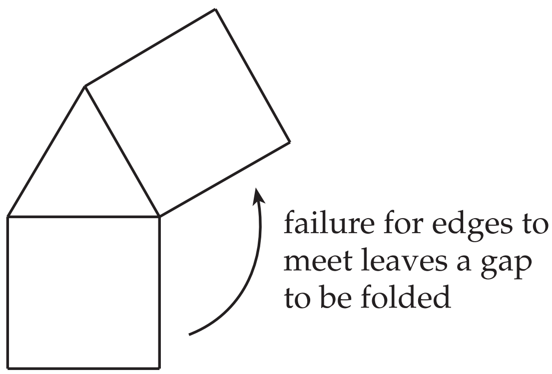
Name ____________________

Partner(s) ____________________

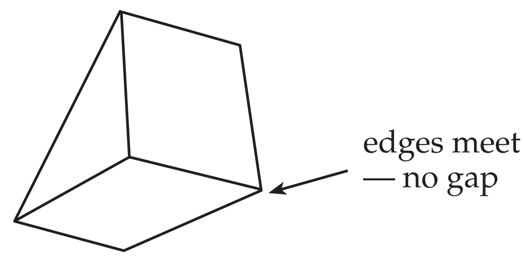
Date ____________________

Part 1: Incomplete Vertex Packings: Folding into 3-Space

We're now going to turn our attention to those configurations of polygons that form incomplete packings around a vertex. For the purposes of analogy, think about an incomplete packing as a failure for the edges of the polygons to meet. When the packing was complete, the edges aligned perfectly. This "failure" for the edges to meet in incomplete packings can be rectified by folding up the configuration so that the edges *do* meet.



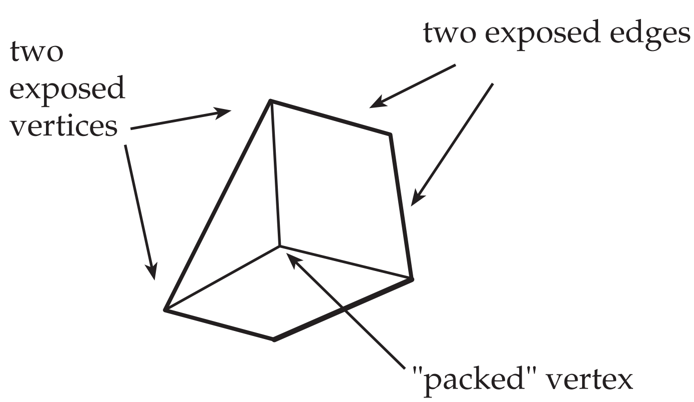
Incomplete Vertex Packing



Folded Vertex Packing

By folding, we have now moved from the plane, a 2-dimensional world, to space, a 3-dimensional world. So this "failure" for the edges to meet actually isn't a failure in a negative sense at all. Rather, it's an opportunity and invitation to explore a new area, namely building objects in 3 dimensions out of polygons.

So what are the interesting questions here? Certainly we can fold any incomplete packings provided that they consist of more than 2 polygons[1], so this single step isn't particularly interesting from a mathematical perspective. Notice however, that once this step is complete, several edges and vertices are still exposed.



[1]You may want to convince yourself that using only 2 polygons won't work.

So here we find an interesting question. If we continue building, so that we repeat the same configuration around each vertex (like we did with a tessellation), can we ultimately produce a shape that perfectly closes in on itself? That is, can we repeat the structure around each vertex so that eventually the edges meet?

These restrictions are quite rigid. Every single vertex must be identical to every other vertex. Furthermore, the edges of each polygon in the structure must match up perfectly with the edges of its neighbors. Can this be done? At this point, take a moment to think about whether there will be many solutions to this problem, relatively few, or no solutions at all. Be sure you can offer some logical basis for your ideas.

We'll begin tackling this question by looking at some specific examples.

1. Consider the packing {4, 3, 4} which has been drawn on page 5-5 of this Exploration.

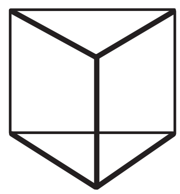
   a. Find a partner. Each of you should cut out one copy of the figure below and fold it into 3-space so that the gap is closed and the edges meet perfectly. Once you've done that, tape your structure together.

   b. Can you see how to put you and your partner's configurations together to make something which closes in to make a solid? You might find that one face overlaps another. This redundancy is fine as long as the faces coincide perfectly.

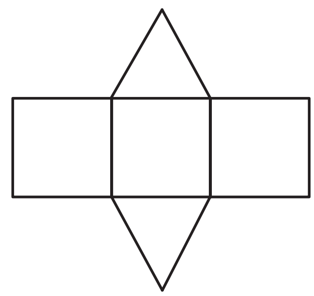
2. You have now created a **polyhedron**, a closed figure whose faces are regular polygons. This particular polyhedron is called a triangular prism. We'll learn more about the names and classes of polyhedra later. But first, using your figure and a good mind, answer the following questions.
   a. Is the configuration at each vertex identical?[2] [If not, redo it so that it is.] A polyhedron that has regular polygons as faces and has the same configuration at each vertex is called a **semi-regular polyhedron**.
   b. How many vertices does your solid have?
   c. What is the take-out angle of the incomplete packing used to create each vertex? That is, before folding, what was the gap?
   d. How many faces does your figure have? How many of these faces are equilateral triangles? How many are squares?

[2] As you examine your prism, keep in mind that {4, 3, 4} is equivalent to {3, 4, 4} and {4, 4, 3}, as the difference in notation is only caused by which face you choose to be the first. Once you pick the starting face, you continue naming the faces either in a clockwise or counterclockwise direction.

3.a. Make sure your solid is taped together well. With scissors, cut along the edges which correspond to the bolded lines in the diagram below. When you're done, you should have a **net**. A net is a single, connected piece that lies flat in the plane. Your net should look like the figure on the right.

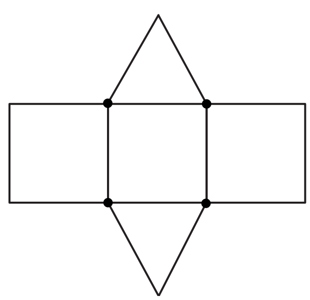


the bold lines show the cuts to be made

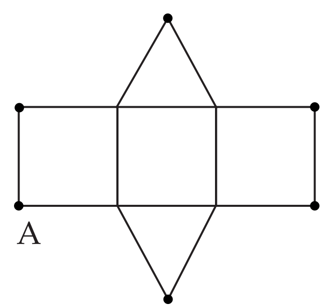


the net

b. A net is one way to look at a polyhedron. It is the solid fully unfolded. A net shows all the faces and, to a large degree, the way the faces are connected. It doesn't keep all of the features of the polyhedron intact however, and it is important to keep in mind that what you see isn't exactly what you get. For instance, in 2b, you should have determined that the right triangular prism has 6 vertices. In the diagram of the net, there are 10 vertices; 4 vertices seem to have the proper vertex configuration [diagram on the left] while 6 vertices don't [diagram on the right].



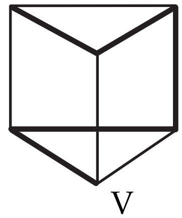
four {4, 3, 4} vertices

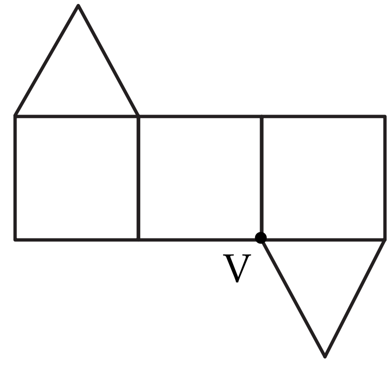


six vertices that don't seem to be {4, 3, 4}

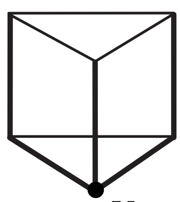
Argue that the vertex marked A in fact has the configuration {4, 3, 4} and that these 6 vertices in the net are actually the 2 other vertices in the solid.

4. Nets are not unique. Shown below on the right is another net for the triangular prism. On the left, the solid is shown with the cuts required to produce the net. One vertex has been labeled to show how the solid and net correspond. Try to visualize the unfolding of the solid into the net. Explain to your partner how it works.





a. Draw the net corresponding to the cutting of the triangular prism indicated by the bolded lines. Label the net so that the correspondence between the vertex V of the solid and the net is clear.

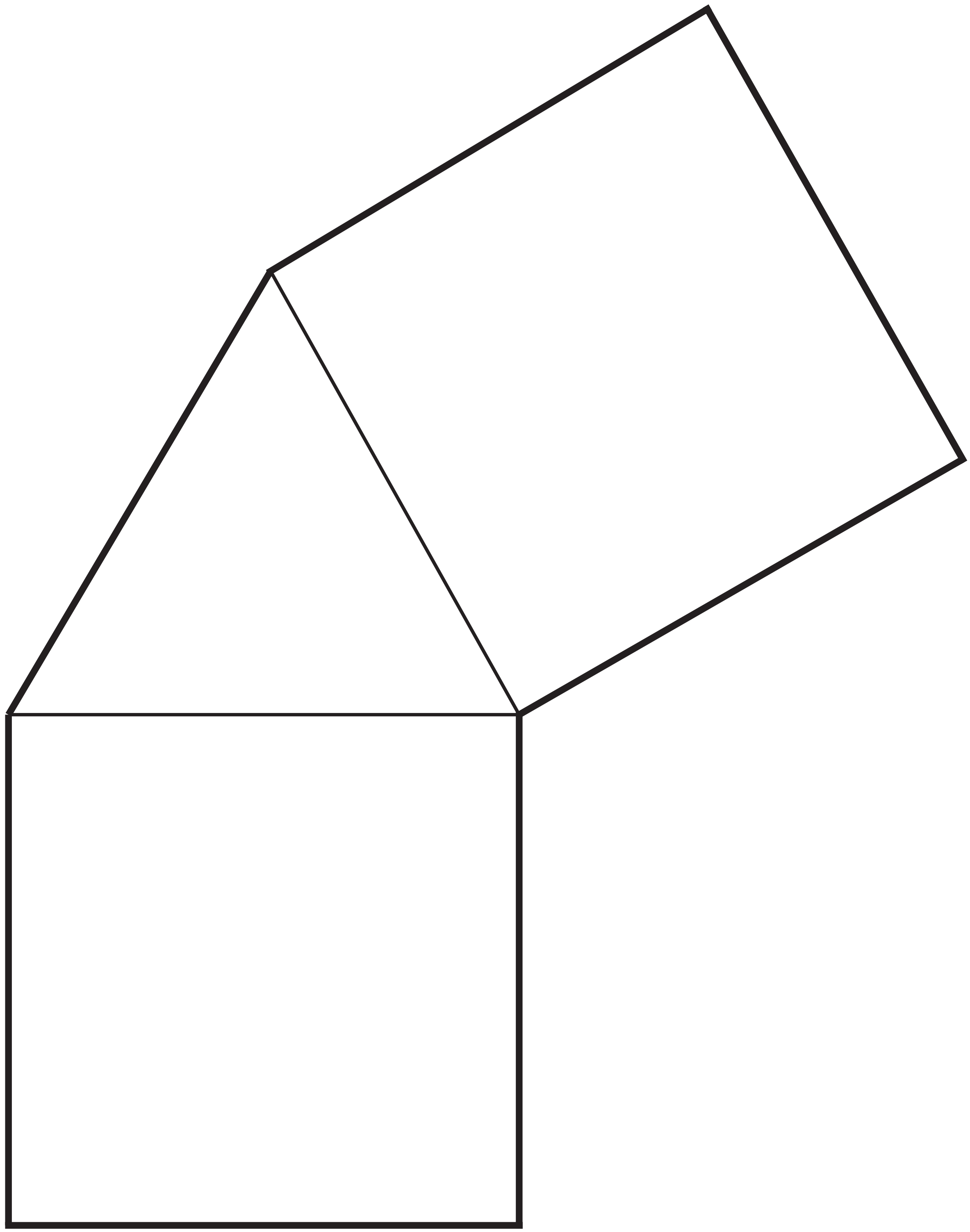


V

net

b. Draw 2 more feasible nets for a right triangular prism. Share them with your partner. Check your partner's nets to be sure they generate the correct solid and that they are different from those already presented.

NET for Problem #1



Polygons and Polyhedra

EXPLORATION 5 (CONTINUED)

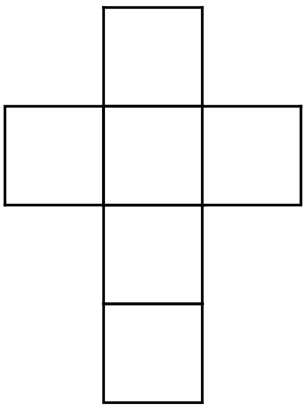
Name ______________________________

Partner(s) ______________________________

Date ______________________________

Part 2: More on Nets

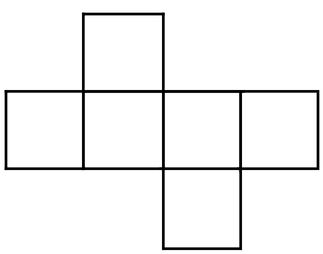
1. Shown below is one net for a cube. A cube is an example of a **regular polyhedron**. Like semi-regular polyhedra, regular polyhedra have the same configuration at each vertex. In addition to having faces which are regular polygons, the faces of a regular polyhedron are all identical.



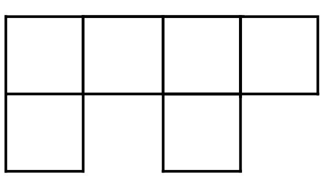
   a. Visualize how this net could be folded to create a cube. What is its vertex configuration?
   b. What is the take-out angle of the incomplete packing?
   c. How many vertices, edges, and faces does a cube have?

2. Shown below are several diagrams. Circle each diagram that is a valid net for a cube and put an X through each of those that could not be folded to create a cube. Be sure you can justify your answers!

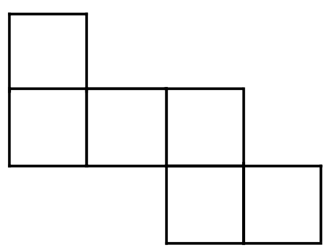
a.



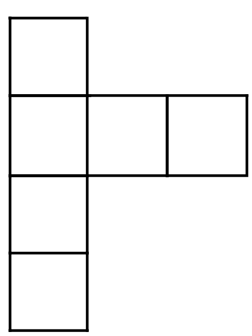
b.



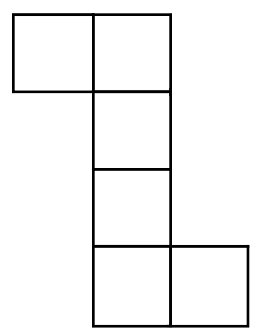
c.



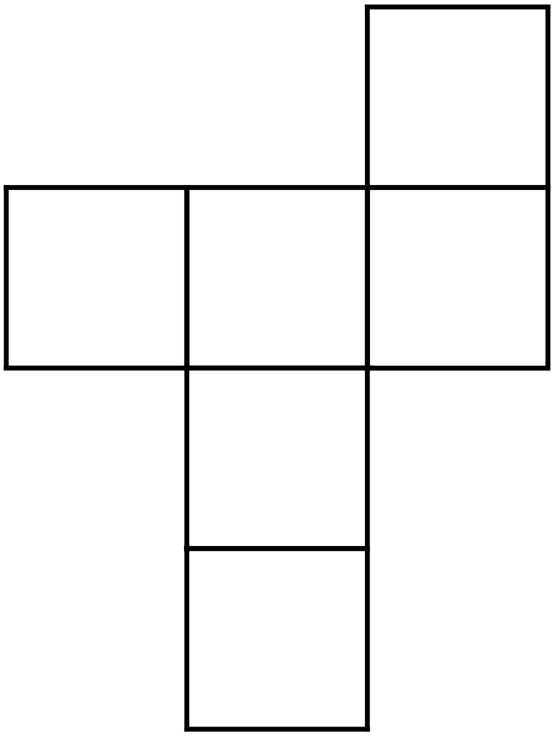
d.



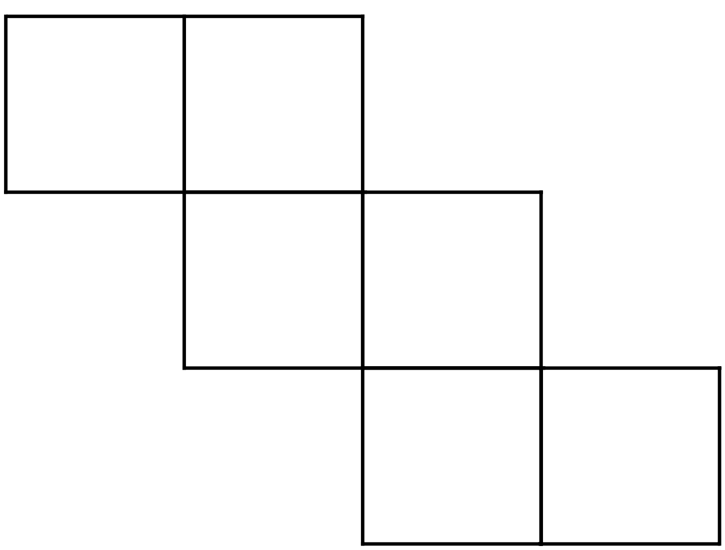
e.



3. Another net for a cube is shown below.
   a. Write the letters C, U, B, and E on this net so that when you fold it, you can properly read the word CUBE around its side.



   b. Write the letters M, A, T, H, I, and E on this net so that when you fold it, you can read the word MATH around its side in one direction and the word TIME around its side in another direction. You will not be able to orient the letters so that both words have all of their letters right-side up. You can get all but one letter properly aligned, however.



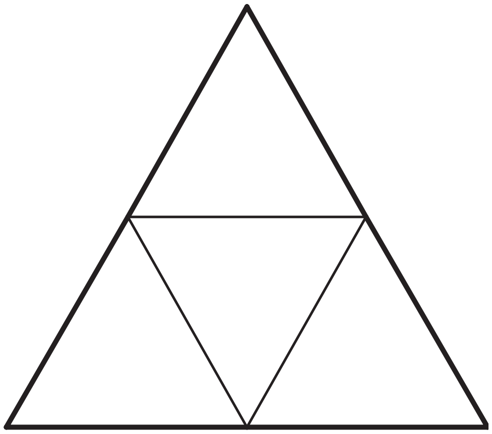
4. On page 5-9 of this Exploration are four nets which may or may not form a closed solid when folded up into 3-space.
   a. Cut out each net by cutting around the outer edges. You may want to leave "tabs" to make taping a little easier. Make creases along the edges, and then try to create a solid by folding the net. If you are successful, tape your figure together.
   b. For each solid you create, examine each of the vertices. Identify those solids that are regular and semi-regular polyhedra.
   c. For each regular or semi-regular polyhedra, determine the vertex configuration, the take-out angle at each vertex, and the total number of vertices.

5. Draw another net for each of the polyhedra you created in #4. If possible, do this without cutting the solids. Ask a partner to check your work.

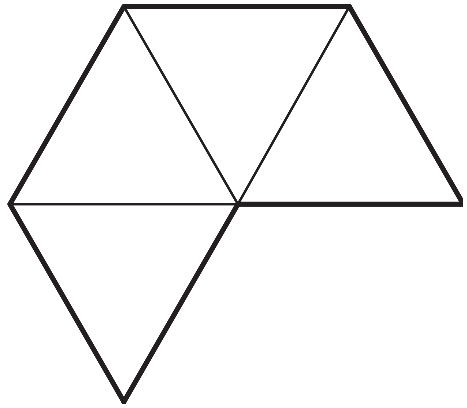
6. Nets are everywhere. Find some package, box, or container around your house or school. Think about what its "net" might look like and then carefully unfold it. You likely won't see a perfect net, as often there is extra material for tabs or material that gets doubled over. Some fun things to try are Chinese food containers, cereal boxes, and cardboard containers from fast food places. Think about the manufacturing process and why the company chose the "net" that they did to produce the package. See what you come up with.

NETS for Problem #4

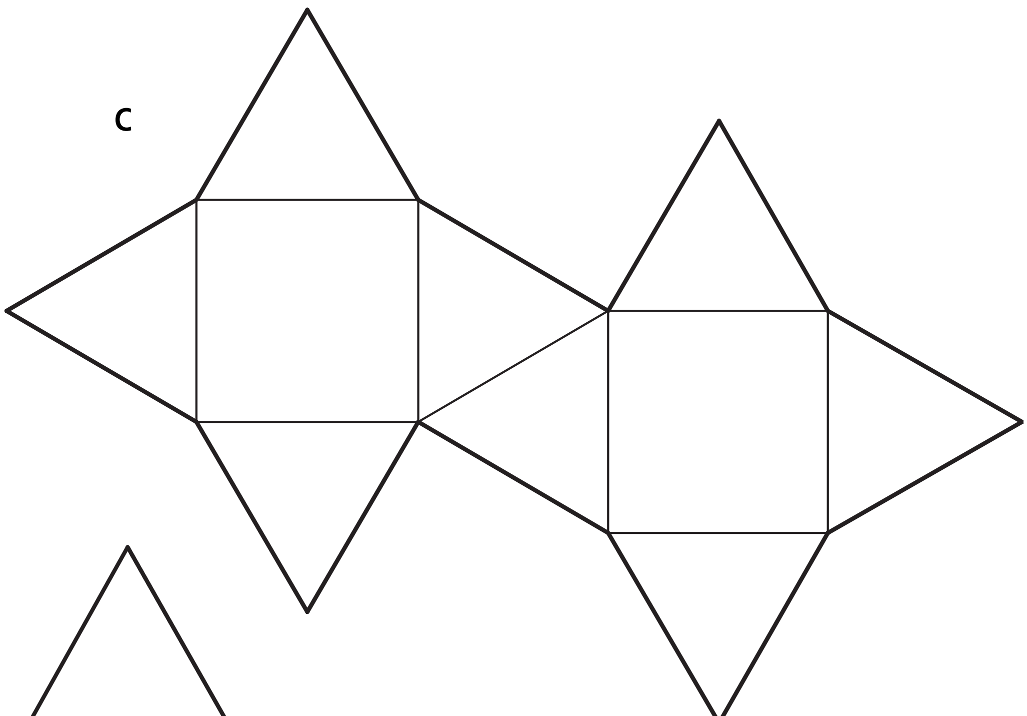
A



B



C



D